INTERNATIONAL JOURNAL ON ECONOMICS, FINANCE AND SUSTAINABLE DEVELOPMENT

IJEFSD

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About Portfolio Theory Models

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A B S T R A C T

The article analyzes the expected return and portfolio risk. The development of a broad and efficient market, a statistical base, as well as rapid progress in the field of computing have led to the emergence of modern theory and practice of portfolio management. We have shown that it is based on the use of statistical and mathematical methods for selecting financial instruments in a portfolio, as well as on a number of new conceptual approaches.

A R T I C L E I N F O

Article history:

Received 29 Oct 2021

Received in revised form 05 Nov 2021

Accepted 10 Dec 2021

Keywords:

Profitability, risk, assets, variance, share of an asset.

Introduction

As a rule, many securities are traded on the financial market: government securities, municipal bonds, corporate shares, etc. If a market participant has free money, then it can be taken to the bank and receive interest or buy securities with it and receive additional income. But which bank should be attributed? What securities should you buy? Low – risk securities are usually low – yield, and high – yield securities are more risky. Economics can provide some guidance to address this issue.

So, an investor is looking for assets in the financial market that can satisfy his wishes regarding profitability and riskiness.

A portfolio is a collection of financial assets held by an investor. It can include both instruments of the

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same type, stocks or bonds, or different assets: securities, financial derivatives, real estate. The main goal of portfolio formation is to strive to obtain the required level of expected return at a lower level of expected risk. This goal is achieved, firstly, due to portfolio diversification, i.e. distribution of investor funds between various assets, and, secondly, careful selection of financial instruments.

The main parameters in portfolio management that an investor needs to determine are his expected return and risk. When forming a portfolio, an investor cannot accurately determine the future dynamics of its profitability and risk.

Therefore, he builds his investment choice on the expected values of return and risk. These values are estimated primarily on the basis of statistical reports for previous periods of time.

Expected portfolio return

An investor’s portfolio consists of several assets, each which its own expected return. What will be the value of the expected portfolio return as a result of their combination? The expected return on a portfolio is defined as the weighted average expected return on its assets, namely:

$$E(r_p) = E(r_1)Q_1 + E(r_2)Q_2 + \ldots + E(r_n)Q_n, \qquad (1)$$

where $E(r_p)$ is the expected return on the portfolio; $E(r_1); E(r_2); \ldots, E(r_n)$ expected profitability of the first, second and n – the assets, respectively; $Q_1, Q_2, \ldots, Q_n$ - share in the portfolio of the first, second, …, n – the assets. The share in a portfolio is calculated as the ratio of its value of the entire portfolio, or:

$$Q_i = \frac{P_i}{P_p}.$$

where Q_i - is the share of the i – the asset; P_i - the value of the asset; P_p - is the portfolio value. The sum of all specific weights included in the asset portfolio is always equal to one:

$$Q_1 + Q_2 + \ldots + Q_n = 1$$

Example 1. A portfolio consists of two assets A and B. $E(r_A) = 8\%$, $E(r_B) = 6\%$ The cost of asset A is \$ 30, asset B is \$ 70. It is necessary to determine the expected return on the portfolio.

Portfolio value is: $\$30 + \$70 = \$100$

$$Q_A = \frac{30}{100} = 0{,}3, \; Q_B = \frac{70}{100} = 0{,}7$$

$$E(r_p) = 8\% \cdot 0{,}3 + 6\% \cdot 0{,}7 = 6{,}6\%$$

The investor will use formula (1) to determine the expected return on the portfolio the expected return on the portfolio based on the expected return on assets. To solve this problem, he must first calculate the expected return on each asset separately. For this you can use the following technique. Suppose, under the conditions it can bring him different results, which at the moment of portfolio formation can only be judged with a certain degree of probability.

Example 2. The profitability of the share is set taking into account the probability:

Profitability (r_i)	5	7	8	10
Probability (π_i)	0,2	0,3	0,3	0,2

Find the expected return on a stock. The expected return on an asset is defined as a weighted arithmetic mean, where the weights are the probabilities of each outcome of an event. In our case, the expected return is:

$$5\% \cdot 0{,}2 + 7\% \cdot 0{,}3 + 8\% \cdot 0{,}3 + 10\% \cdot 0{,}2 = 7{,}5\%$$

Let's write down the formula for determining the expected return on an asset in general form:

$$E(r) = \sum_{i=1}^{n} r_i \cdot \pi_i$$

Where $E(r)$ - is expected return on the asset, r_i- is the return on the asset in the i – th case (implementation of the experiment); π_i - is the probability of the i – th outcome, n – is the number of possible values of the return. Note that, if $\pi_1 = \pi_2 = \ldots = \pi_n = \frac{1}{n}$, then $E(r)$ is the arithmetic mean and is denoted by the symbol $\bar{r}$.

Expected risk of an asset. When purchasing an asset, an investor focuses not only on the value of its expected return, but also on the level of its risk. The expected return acts as a certain value that the investor hopes to receive, for example 7,5%. The possibility of obtaining this result is confirmed by the previous dynamics of the asset's return. However, 7,5% is only an average. In practice, the return that the investor will receive may be equal to or different from 7,5%. Thus, the investor's risk is that he may get a result that is different from the expected return strictly speaking, the risk of an investor is that he will receive worse than the expected result i.e. its yield will be less than 7,5%. If the actual profitability turns out to be more than 7,5%, then this is a plus for the investor. In practice, indicators of variance and standard deviation are used as a measure of risk. They show to that extent and with what probability the actual return on an asset may differ from the value of its expected return, that is, the average return.

These parameters take into account deviations both in the direction of increasing and decreasing profitability in comparison with the expected value. as we noted above, the actual risk is that the actual profitability will be lower than the expected one, however, the noted parameters are used as a measure of risk, primarily due to the simplicity of their determination.

The variance of the asset's return is determined is determined by the formula

$$\sigma^2 = \sum_{i=1}^{n} \pi_i \left(r_i - E(r)\right)^2$$

with $\pi_1 = \pi_2 = \ldots = \pi_n = \frac{1}{n}$, the last formula takes the form:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} \pi_i \left(r_i - \bar{r}\right)^2$$

where σ^2- is the sample variance of the asset's return; n is the number of observation periods; r_i- the return on the asset in the i – th period, $\bar{r}$ - the average return on the asset:

$$\bar{r} = \frac{r_1 + r_2 + \ldots + r_n}{n}$$

The standard deviation is defined as the square root of the variance

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$$\sigma_0 = \sqrt{\sigma^2}$$

Expected portfolio risk

The expected risk of a portfolio is a combination of the standard deviations of its constituent assets. However, unlike the expected return on a portfolio, its risk is not necessarily the weighted average of the standard deviations of asset return.

To determine the degree of relationship and the direction of change in the returns of two assets, such indicators as covariance and correlation coefficient index is determined by the formula.

$$\operatorname{cov}(r_A, r_B) = \frac{1}{n}\sum_{i=1}^{n}\left(r_{A_i} - \overline{r_A}\right)\left(r_{B_i} - \overline{r_B}\right),$$

where $\operatorname{cov}(r_A, r_B)$ - is the sample covariance of the returns on assets A and B, with $\overline{r_A}$ and $\overline{r_B}$ are the average returns on assets A and B, in the i – th period, respectively.

Positive covariance means that asset returns move in one direction in one direction, negative in the opposite direction.

Example 3. The portfolio consists of two assets A and B and the following table shows the values of their returns for 4 years:

year	1	2	3	4
Profitability A	0,1	0,16	0,14	0,17
Profitability B	0,12	0,18	0,14	0,15

For our example

$$\overline{r_A} = 0{,}1425, \quad \overline{r_B} = 0{,}1475, \quad \operatorname{cov}(r_A, r_B) = \frac{0{,}0018248}{4} = 0{,}0004562$$

Another indicator of the degree of relationship between changes in the returns of two assets is the correlation coefficient, which is calculated by the formula

$$r_{12} = corr(r_A, r_B) = \frac{\operatorname{cov}(r_A, r_B)}{\sigma_A \cdot \sigma_B} \qquad (2)$$

where σ_A and σ_B are the standard deviations of the returns on assets A and B, respectively.

Portfolio risk with two assets

The risk of a portfolio consisting of two assets is calculated using the formula

$$\sigma_p^2 = \theta_A^2\sigma_A^2 + \theta_B^2\sigma_B^2 + 2\theta_A\theta_B \operatorname{cov}(r_A, r_B) \qquad (3)$$

Based on formula (2), formula (3) can be written as:

$$\sigma_p^2 = \theta_A^2\sigma_A^2 + \theta_B^2\sigma_B^2 + 2\theta_A\sigma_A\theta_B \cdot \sigma_B corr(r_A, r_B) \qquad (4)$$

suppose $r_{12} = 1$ i.e., there is a direct functional relationship between the returns on assets A and B. Then

$$\sigma_p^2 = \theta_A^2\sigma_A^2 + \theta_B^2\sigma_B^2 + 2\theta_A\sigma_A\theta_B \cdot \sigma_B = \left(\theta_A\sigma_A + \theta_B\sigma_B\right)^2, \quad \text{i.e.}$$

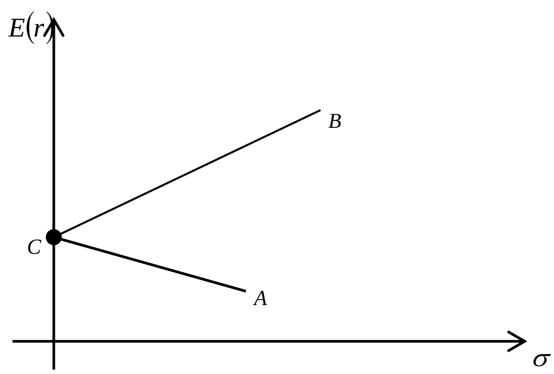
$$\sigma_p = \theta_A\sigma_A + \theta_B\sigma_B \quad (5)$$

From formula (5) it follows that if the return on assets have a correlation of +1, then the portfolio risk is the weighted average risk of the assets included in it. Combining such assets into one portfolio does not allow taking advantage of diversification opportunities to reduce risk, since when the market situation changes, their retunes will change in direct proportion in the same direction. In this case, diversification does not reduce the risk, but only averages it. Now suppose -1. This means that there is a linear functional relationship with a negative coefficient between the returns r_A and r_B. In this case, the formula is as follows.

$$\sigma_p^2 = \theta_A^2\sigma_A^2 + \theta_B^2\sigma_B^2 - 2\theta_A\sigma_A\theta_B \cdot \sigma_B = \left(\theta_A\sigma_A - \theta_B\sigma_B\right)^2, \quad \text{i.e.}$$

$$\sigma_p = \theta_A\sigma_A - \theta_B\sigma_B \quad (6)$$

Combining into a portfolio of assets with a correlation of -1 makes it possible to reduce its risk in comparison with the risk of each individual asset, since when the market conditions change, the oppositely directed movements in the yield of assets *A* and *B* will cancel each other out. In this case, the expected return on the portfolio will remain unchanged and will depend on the expected return on each asset and its share in the portfolio. By combining assets *A* and *B* in a portfolio in different proportions, an investor has the opportunity, in terms of risk and return, to form any portfolio that will lice on lines *AC* and *CB*, as shown in fig.1.



options for portfolios consisting of two assets *A* and *B* with a correlation of return -1

fig. 1.

At a point, the investor’s portfolio will have no risk. To form such a portfolio, it is necessary to find the corresponding shares of assets *A* and *B*. to do this, we equate the right side of equation (6) to zero and define θ_A and θ_B:

$$\sigma_p = \theta_A \cdot \sigma_A - \theta_B \cdot \sigma_B = 0$$

since

$$\theta_A = 1 - \theta_B,$$

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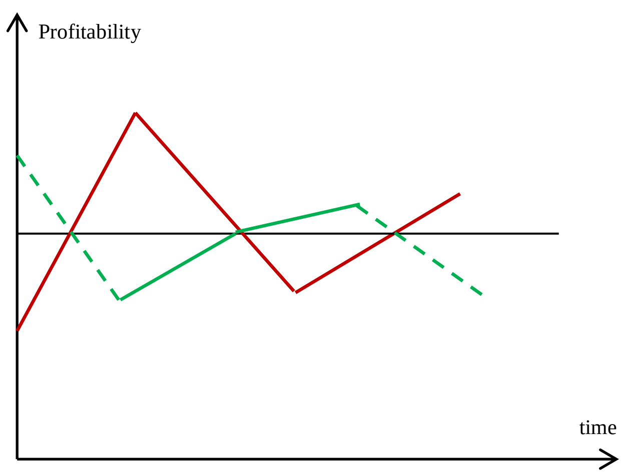
then $$(1-\theta_B)\cdot\sigma_A-\theta_B\cdot\sigma_B=0$$

from here

$$\theta_A=\frac{\sigma_B}{\sigma_A+\sigma_B},\qquad \theta_B=\frac{\sigma_A}{\sigma_A+\sigma_B}$$

Thus, it is always possible to create a risk-free portfolio from two assets with a complete negative correlation of return. The return of such a portfolio depending on time is shown in fig.2. with a horizontal line. The other two lines show an example of a possible change in the returns on the assets included in the portfolio. *A* drop in the profitability of one asset is always fully compensated by an increase in the profitability of another asset. Obviously, by managing these assets separately, it would be possible to increase profits – it is enough to sell them at the tops and buy at the troughs. These are the costs of diversification. However, active management is associated with increased risk, requires costs for predicting market behavior and is not always successful



return on two assets with complete negative correlation

fig. 2.

Dominant portfolio. Portfolio combinations for other values of the yield correlation are located within the triangle. Thus, the space of the triangle *ABC* represents all possible combinations of risk and return of portfolios consisting of two assets within the limits of their return correlation from -1 to +1.

At the same time, in practice, the overwhelming majority of assets have a correlation other than -1 and 1, and most assets have a positive correlation. If you build a graph for portfolios consisting of assets *A* and *B* with less correlation than +1, then it will take on a convex form.

As shown in Figure, if the assets have c correlation less than +1, then the investor can form any portfolio that would be located on the *ADB* curve. However, the rational investor will make his choice only on the upper part of this curve, namely, the segment *DB*, since it contains portfolios that bring a higher level of expected return at the same risk compared to portfolios in the segment *DA*. Let's compare the portfolios p_1 and p_2 for clarity. Both portfolios have a risk equal to σ_1, but the expected return on portfolio p_2 is greater than the expected return on portfolio p_1 .

If one portfolio has a higher level of return at the same level of risk or a lower risk at the same rate of return than other portfolios, then it is called dominant. So, in Fig.3, portfolio p_2 is will be dominant in relation to portfolio p_1, since both of them have the same risk (σ_1), but the return on the portfolio

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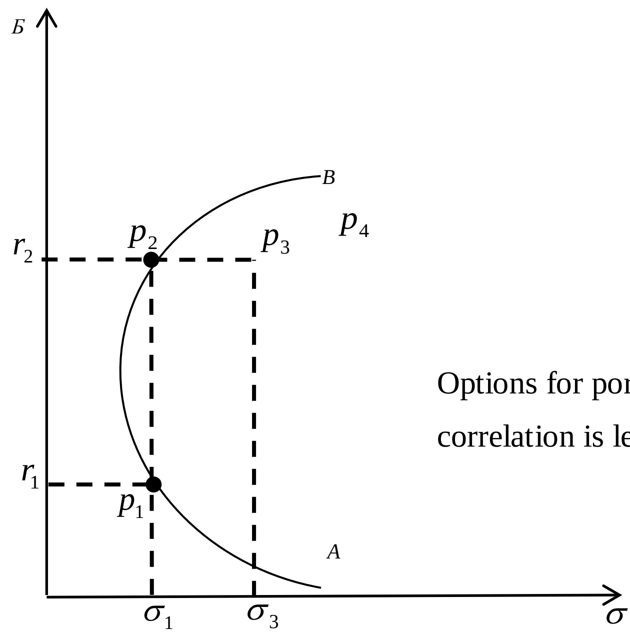
$p_2\ (r_2)$ is greater them the return on the portfolio $p_1\ (r_1)$. Similarly, portfolio p_2 will dominate over portfolio p_3, since they both have the same return (r_2), but portfolio $p_2\ (\sigma_2)'$s risk is less than p_3's. At the same time, if we compare p_1 and p_4, then we cannot say that one of them is dominant in relation to the other, since they have different values of both the expected return and risk. Portfolio p_4 has both higher expected return and higher risk than portfolio p_1. The rational investor will always opt for the dominant portfolio because it is the best choice in terms of return and risk for all possible alternatives for other portfolios.



Options for portfolios of two assets, the correlation is less +1

Fig. 3.

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