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MODERN METHODS OF PROOF OF INEQUALITIES

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ABSTRACT

The article presents new effective methods of proving inequalities and problems in various mathematical Olympiads on their application. The manual is intended for gifted students of general secondary schools, academic lyceums and vocational colleges, teachers of mathematics and students of pedagogical universities. One of the basic procedures for proving inequalities is to rewrite them as a sum of squares and then according to the most elementary property that the square of real number is non negative to prove a certain inequality.

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INTRODUCTION

In math classes, students learn to draw conclusions independently from the first lesson. The task of a mathematics teacher is to develop students' interest in learning mathematical laws while developing logical thinking skills. One of the main tasks of the school is to prepare students for logical thinking, research, creativity, independent learning and self-development.

In addition to imparting knowledge to students in the process of proving inequalities, they have a great opportunity to develop their abilities, qualities such as diligence, will and character.

Proof of inequalities in knowledge testing allows students to think about the development of thinking, the right choice of actions, the right choice of calculation skills, and the right choice of actions. The article presents new effective methods of proving inequalities and problems in various mathematical Olympiads on their application. The manual is intended for gifted students of general secondary schools, academic lyceums and vocational colleges, teachers of mathematics and students of pedagogical universities.

1. Prove that for any real number x the following inequality holds.

$x^{12} - x^9 + x^4 - x+1>0.$

Solution. We consider two cases $x <1$ and $x \geq 1$

Let $x <1$; We have

$x^{12}-x^9+x^4-x+1=x^{12}+(x^4-x^9)+1-x$

Since $x<1$ We have $1-x>0$ and $x^4>x^9$; i.e $x^4-x^9>0$ so in this case

$$x^{12}-x^9+x^4-x+1>0$$

i.e. the desired inequality holds.

For $x\geq 1$ we have

$x^{12}-x^9+x^4-x+1=x^8(x^4-x)+(x^4-x)+1=(x^4-x)(x^8+1)+1=$

$=x(x^3-1)(x^8+1)+1$

Since $x\geq 1$ we have $x^3\geq 1$, i.e. $x^3-1\geq 0$

Therefore

$$x^{12}-x^9+x^4-x+1>0$$

and the problem is solved.

2. If $a,b,c\in R$, Prove the inequalities

$$3(ab+bc+ca)\leq (a+b+c)^2\leq 3\left(a^2+b^2+c^2\right)$$

Solution. We have

$3(ab+bc+ca)=ab+bc+ca+2(ab+bc+ca)\leq a^2+b^2+c^2+2(ab+bc++ca)=(a+b+c)^2=$

$=a^2+b^2+c^2+2(ab+bc+ca)\leq a^2+b^2+c^2+2(a^2+b^2+c^2)=3(a^2+b^2+c^2)$.

Equality occurs if and only if a=b=c.

3. Let $a,b,c\in R$. Then

$(a-c)^2\leq 2(a-b)^2+2(b-c)^2$.

Proof. We have

$$(a-c)^2\leq 2(a-b)^2+2(b-c)^2$$
$$a^2-2ac+c^2\leq 2\left(a^2-2ab+b^2\right)+2\left(b^2-2cb+c^2\right)$$
$$\leftrightarrow a^2+4b^2+c^2-4ab-4bc+2ac\geq 0$$
$$\leftrightarrow (a+c-2b)^2\geq 0$$

which clearly holds.

4. Let $a\geq b\geq c$ Then:

$$(a-c)^2\geq (b-c)^2+(a-b)^2$$

Proof. We have

$$(a-c)^2\geq (b-c)^2+(a-b)^2$$
$$a^2-2ac+c^2\geq \left(b^2-2bc+c^2\right)+\left(a^2-2ab+b^2\right)$$
$$\leftrightarrow b^2+ac-ab-bc\leq 0$$
$$\leftrightarrow (b-a)(b-c)\leq 0$$

which is true since $a\geq b\geq c$.

5. Let $a,b,c\geq 0$. Prove the inequality

$$a^3+b^3+c^3\geq 3abc$$

Solution. We have

$$a^3+b^3+c^3-3abc=\frac{a+b+c}{2}\left((a-b)^2+(b-c)^2+(c-a)^2\right)\geq 0$$

which is obviously true.

6. Let $a\geq b\geq c$. Then $\frac{a-c}{b-c}\geq \frac{a}{b}$.

Proof. $\frac{a-c}{b-c}\geq \frac{a}{b}\leftrightarrow b(a-c)\geq a(b-c)$

$ac\geq bc \leftrightarrow a\geq b$

Here we will introduce the reader to the simplest and most often used forms which are as follows:

1. $a^2+b^2+c^2-ab-bc-ac=\frac{(a-c)^2+(b-c)^2+(a-b)^2}{2}$
2. $a^3+b^3+c^3-3abc=(a+b+c)\left(\frac{(a-c)^2+(b-c)^2+(a-b)^2}{2}\right)$
3. $a^2b+b^2c+c^2a-ab^2-bc^2-ca^2=\frac{(a-c)^3+(b-c)^3+(a-b)^3}{3}$

4. $$a^3+b^3+c^3-a^2b-b^2c-c^2a=\frac{(2a+b)(a-c)^2+(2b+c)(b-c)^2+(2c+a)(a-b)^2}{3}$$

5. $$a^3b+b^3c+c^3a-ab^3-bc^3-ca^3=(a+b+c)\left(\frac{(a-c)^3+(b-c)^3+(a-b)^3}{3}\right)$$

6. $$a^4+b^4+c^4-a^2b^2-b^2c^2-c^2a^2=\frac{(a+c)^2(a-c)^2+(b+c)^2(b-c)^2+(a+b)^2(a-b)^2}{2}$$

REFERENCES:

- "Inequalites. Theorems, Techniques and Selected problems" (Zdravko Cvetkovski).
- "Mental Arithmetics Trics" (Andreas Klein).
- "Secrets of mental math" (Arthur Benjamin and Michael Shermer).
- Kim Hung, P: "Secrets in Inequalities", Gil publishing House (2007).
- Andreescu, T., Enescu, B.: Mathematical Olympiad Treasures. Birkhäuser, Basel (2003)
- Andreescu, T., Cirtoaje, V., Dospinescu, G., Lascu, M. : Old and New Inequalities. Gil Publishing House, Zalau (2004)

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