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SOLVING THE EQUATION OF HEAT DISSIPATION IN A ROD BY THE FINITE ELEMENT METHOD

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Abstract: The designer can only achieve the requisite thermal performance of external enclosing structures if they have a thorough understanding of the processes that occur in fences during heat transfer and the ability to employ the right calculations. As you may be aware, a thorough analytical representation of the heat conduction process comprises a differential equation and specified boundary conditions.

Keywords: Numerical method of analysis, finite difference method, equation, finite element method, boundary-value differential problems, coordinate functions

Introduction

The use of computational techniques and numerical methods significantly expands the classes of the studied field problems of heat transfer, allowing one to obtain approximate solutions of multidimensional, nonlinear, nonstationary problems for which the use of exact and approximate analytical methods is not possible. When choosing mathematical models that describe heat transfer processes in real objects, the boundaries of their permissible complexity are currently often determined not so much by the capabilities of numerical methods and computer resources, as by the lack of reliable input information for these models. Methods of numerical solution It is important to solve boundary value issues for differential equations in partial derivatives in given areas of variation of spatial variables and time intervals while identifying various space-time fields. The discretization of the spatial and temporal domains at the very first stage of development is a distinguishing aspect of the application of numerical approaches. At the second stage, a system of algebraic equations is drawn up for the values of the sought functions at these nodal points. On the third, the solution of the system is carried out, and the values of the investigated quantities at the nodal points are found. Note that the discretization of the region is often done in the calculation based on analytical solutions, but in these cases it is carried out at the final stages, which are realized after the analytical solution is obtained.

There are two main numerical methods for solving partial differential equations: the finite difference method and the finite element method. They differ in the methods of obtaining a system of equations for the values of the sought functions at the nodal points. The finite difference method is based directly on the differential equation and boundary conditions, and the finite element method is based on the equivalent variational formulation of the problem.

The finite difference method. The numerical method of analysis, called the finite difference method, has a long history of development. It was originally used for manual counting. However, due to the low productivity of calculators, this method was used relatively recently only for approximate calculations, when analytical methods of solving turned out to be completely unacceptable. The advent of computers with high performance and a significant amount of RAM has brought finite-difference methods to the forefront of modern methods for solving thermophysical problems. The idea behind the finite difference method (FCD) is as follows. The area of continuous change of arguments is replaced by a computational grid - a discrete set of points (nodes). Instead of a function of continuous arguments, functions of discrete arguments are introduced - grid functions defined at the grid nodes. The partial derivatives included in the differential equation and boundary conditions are replaced (approximated) by difference relations. As a result of this substitution, the boundary value problem in partial derivatives is reduced to a system of difference (algebraic) equations, called the difference scheme. If the solution

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of the system of difference equations exists and when the grid is crushed, it tends to solve the problem (i.e., converges), then it is the desired numerical solution of the boundary value problem.

Main part

When developing a finite-difference analog of a specific heat and mass transfer problem, it is necessary to consider how to choose a grid and build a difference scheme, determine the accuracy of the approximation of the original problem by the difference scheme, check the stability of the difference scheme, and find out the rate of convergence of the solution of the difference problem to the solution of the original heat and mass transfer problem. Finite element method. We obtained the systems of algebraic equations for determining the numerical solution by replacing the derivatives in the differential equation and in the boundary conditions or in the heat balance equations for unit cells by finite differences. Thus, in the finite difference method, the starting point for obtaining an approximate solution is a differential boundary value problem. However, the required field can also be found from the solution of the corresponding variational problem. Its numerical solution is based on the widely used finite element method (FEM) [7,27]. Let us consider the essence of the FEM using the example of solving a two-dimensional stationary heat conduction problem in a region D of arbitrary shape (Figure 1):

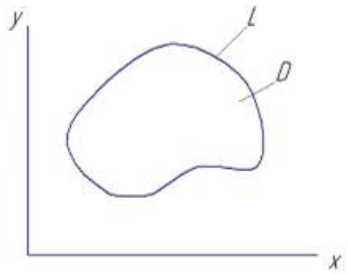
$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+q_v=0 \quad (1.1)$$

Under boundary conditions of the third kind:

$$\left[\lambda\frac{\partial T}{\partial n}+\alpha T\right]_L=q_s \quad (1.2)$$

where λ - thermal conductivity; α is the heat transfer coefficient at the boundary; qv, qs - volumetric and surface power density of heat sources.

The values λ, α, qv, qs can be specified in the form of arbitrary functions of coordinates x, y, including piecewise functional functions.



Picture 1. Two-dimensional stationary heat conduction problem in domain D.

The finite element method is based on determining the temperature field by an approximate solution of the corresponding variational problem. To formulate this problem, we recall the concept of a functional. An operator I [f (x)] is called a functional defined on a certain set of functions if each function f (x) from this set is assigned a numerical value I [f (x)] according to some rule. In other words, a functional is, as it were, a "function from a function". In practical applications, one usually encounters functionals defined in the form of certain integrals, whose integrands include functions f (x). Many boundary-value differential problems of heat conduction and convective heat transfer are equivalent to the problems of finding functions that provide a minimum to some specially constructed functionals. The problem of finding a function that minimizes a functional is called a variational one. Many approximate analytical methods for solving heat conduction problems have been developed on the basis of the transition from boundary-value differential problems to variational ones. Note that the possibility of a variational formulation of the problem of determining the temperature field (1.1), (1.2) is due to the properties of the differential operator of the heat equation.

We give a variational formulation of the considered boundary value differential problem (1.1.1), (1.1.2) without proof. The problem of solving equation (1.2) with boundary conditions (1.1.2) is equivalent to the problem of determining the function T (x, y) minimizing the functional I [T (x, y)] of the form

$$I[T(x,y)]=\iint_D\left[\lambda\left(\frac{\partial T}{\partial x}\right)^2+\lambda\left(\frac{\partial T}{\partial y}\right)^2-2q_vT\right]dxdy+\int_L(\alpha T^2-2q_sT)dl$$

(1.3)

The most widespread technique for obtaining an approximate solution of variational problems is as

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follows. An approximation for the required function T (x, y) is sought in the form:

$$T(x,y) \approx \sum_{m=1}^{M} a_m f_m(x,y) \quad (1.4)$$

where am are unknown constant coefficients and fm (x, y) are known functions of spatial coordinates. If we substitute (1.4) into functional (1.3), then we can carry out the integration over the spatial variables and obtain the value I, which depends not on the unknown function, but on the unknown coefficients of expansion (1.4).

$$I = I(a_1, \ldots, a_M) \quad (1.5)$$

It is obvious that to determine the approximate solution of the variational problem in the form of (1.4), one should find the values a1..., aM that provide the minimum of the functions (1.5), i.e., the problem is reduced to finding the minimum point of the "ordinary" function of several variables. As is known, the values a1..., aM, which provide the minimum of the function I, are found from the solution of the system of equations:

$$\frac{\partial I}{\partial a_1} = 0, \ldots, \frac{\partial I}{\partial a_M} = 0 \quad (1.6)$$

Having solved the system (1.6), one can find the values a1..., aM, and substituting them in (1.4), determine the approximate solution of the variational problem.

The central point in the described method is the assignment of the coordinate functions of the decomposition (1.4) I1...,IM. The finite element method is based on the use of the described approximate solution scheme with a specific choice of the type of coordinate functions f1..., fM. Thanks to this choice, the unknown coefficients in the decomposition (1.4) acquire a clear physical meaning.

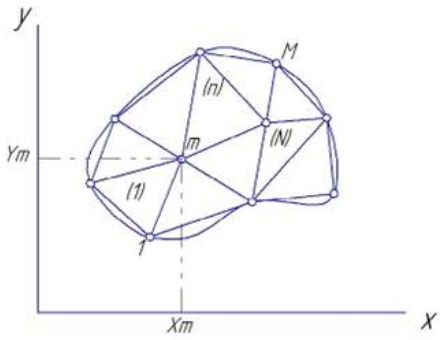

Figure 2. Partitions of the domain of definition of the desired continuous value.

The construction of coordinate functions is carried out in the FEM after dividing the domain of definition of the desired continuous value into N subdomains, called elements, and fixing in them M nodal points selected at the boundaries of the elements (Figure 1.5). Note that the number of terms in the expansion is equal to the number of nodal points. Each of the functions fm (x, y) has the following properties. The value of the function fm (x, y) at the m-th nodal point with coordinates x = xm, y = ym is equal to one, and at the other nodal points it is zero. In addition, the function Im (x, y) can be nonzero only in elements containing the mth node. In the rest of the area D, it is considered equal to zero.

With this choice of coordinate functions fm (x, y), any unknown coefficient am in the expansion is equal to the approximate value of the temperature am at the mth nodal point. Indeed, when setting the coordinates of the m-th node in the approximation (x = xm, y = ym), the values of all coordinate functions, except for the m-th function, will be equal to zero, which means that the m-th function is one, and therefore:

$$T(x_m, y_m) \approx \sum_{m=1}^{M} a_m f_m(x_m, y_m) = a_m f_m = a_m$$

When using the expansion at each point of the domain D, only those coordinate functions "work" for which the coefficients are equal to the approximate values of the temperatures of the nodal points of the finite element containing the given point.

We note two important facts that follow from the considered properties of the coordinate functions.

First, the function I (a1 ..., aM), obtained by substituting the expansion into the functional, will be a function of the unknown approximate temperature

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values at the nodal points u1 ..., uM, and the equations following from the minimum condition (1.6),

$$\frac{\partial I}{\partial u_1} = 0, \ldots \frac{\partial I}{\partial u_M} = 0$$

They are algebraic equations of the finite element method difference scheme with respect to the required temperatures at the nodes.

Second, the spatial distribution of temperature inside any element is approximated by the sum of the products of coordinate functions by coefficients equal to the approximate temperature values at the nodal points belonging to this element.

Coordinate functions fm (x, y), m = 1, M are constructed on the basis of the so-called shape functions of elements. Each of the shape functions of a particular element is equal to one at one "own" nodal point belonging to the given element, and zero at the other nodes of this element, i.e. as many form functions are entered for an element as there are nodes. Outside an element, all of its form functions are assumed to be zero.

Conclusion

Thus, the shape function of the n-th element, equal to one at the m-th point belonging to it, is a "representative" of the coordinate function fm (x, y) in this n-th element. Therefore, the temperature field in the n-th element is approximated by the sum of the products of its shape functions by the approximate values of temperatures at its nodal points. Obviously, for each element, its own approximation is obtained, but the continuity of the temperature field should be preserved at the boundaries of the elements.

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