# Mathematical Definition of Cutting Ability of Knives 

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#### Abstract

The article covers an analytical expression for calculating the cutting ability of thin lamellar knives, considering the characteristics of longitudinal and transverse microrelief of blade and cutting modes. The main parameters of microgeometry that determine the cutting ability of blades are the width of the cutting edge, the height and longitudinal pitch of the micro-teeth, and the characteristics of the reference curve.


Keywords: modeling of micro-teeth, cutting edge, blade, working height of the knife, microgeometry, cutting ability, sliding cutting, working height of a tooth

## Introduction

High-quality sharpening of thin lamellar knives at finishing the cutting edge, using multi-pass grinding with a small depth of cut and the use of wheels from elbor provides a very small blade thickness and the location of micro-teeth on the edge not in two parallel rows [1,2], but in one a row with practically zero transverse step.

The results obtained allow abandoning the modeling of micro-teeth in the form of pyramidal or conical protrusions with rounded tops located on the cutting edge with a certain transverse and longitudinal pitch, and represent each micro-tooth of the blade as a truncated cylinder (Fig. 1.a) with small radius of elliptical section and height (generatrix) in the narrow part, equal to the thickness of the cutting edge (Fig. 1.b.). Such a model corresponds to a greater extent to the real microtopography of the cutting edge, corresponds to the mechanism of blade
wear and allow taking into account such important parameters of the cutting edge as the characteristics of the straight section of the curved support surface, which play a major role in the period of normal wear of the knife.


Fig. 1. a) interaction of the micro-tooth of the cutting edge with the material being cut, b) model of the micro-tooth of the blade

## Main part

Thereby, the cutting edge is a rough surface. By the characteristics of the roughness, we can judge the cutting properties of the lamellar knife [2].
Based on the theory of interaction of rough surfaces [3], we represent the cutting edge of the blade in the form of set of truncated cylindrical cusps located in height with a certain density, the distribution law of which is expressed by a function $\eta=b \cdot \varepsilon^{\nu+1}$ at the first stage and a function; at the second stage of the knife operation $h=k(1-\varepsilon)+c$.
We will assume that the distribution of microroughness vertexes along the height for the proposed model and the real cutting edge are described by the same functions, and also that the same contact area, load and friction force correspond to the same value of the approach of the real edge and the model. The cutting edge moves relative to the material at a speed $u_{1}$, penetrating into it at a speed $u_{2}$. In this case, the micro-teeth interact with the volume of the material:

$$
\begin{equation*}
V=F_{\Phi} \frac{h b_{\varepsilon}^{\nu+1}}{v+1} \tag{1}
\end{equation*}
$$

where: $F_{\phi}$ is actual contact area;
$h$ is the working height of the micro-teeth;
$b, v$ are parameters of the curved support surface;
$\varepsilon$ is relative approach; $\varepsilon=h / R_{\text {max }}$
$R_{\text {max }}$ is the maximum height of the micro-teeth.
Cutting capacity $Q$ can be represented as the ratio of the penetration depth $H$ of the knife to the cutting path $Z$ :
$Q=H / Z$
The ratio of the feed speed $u_{l}$ to the cutting speed $u_{2}$
$Q=u_{1} / u_{2}$
Where: $u_{1}$ is specified, and the value of $u_{2}$ is obtained as a result of the operation of the blade with its specific characteristics set in advance, i.e. cutting speed and efforts $R_{2}$. The cutting ability of a knife can be determined experimentally, provided that $R_{2}=$ const, $u_{1}=$ const. The less the path of interaction of the cutting edge with the material being cut, the higher the cutting ability will be.
$Q=\frac{F_{\Phi}}{F_{a}}$
(4)
$\underline{F_{a}}$ is contour contact area $F_{a}=a l_{b} ; a$ is blade width; $l_{b}$ is base length; $q$ is the Kragelsky criterion [4], $q=\frac{V}{F_{\phi} \cdot d} ; d$ is the diameter of the micro-serration section.
Substituting (1) into (2), and then (3) into (4), after simplifications, we obtain:

$$
\begin{equation*}
Q_{1}=\frac{h b \varepsilon^{v+1}}{d(v+1)} \cdot \frac{F_{\phi}}{F_{a}} \tag{5}
\end{equation*}
$$

This expression is valid for the initial stage of the blade's operation, when the first section of the supporting surface has a curvilinear character. Further, micro-teeth of the second rectilinear section are included in the work. Therefore, the expression for the cutting ability in this period of work according to a similar scheme takes the form:
$Q_{2}=\frac{h[k(1-\varepsilon)+c]}{d} \cdot \frac{F_{\phi}}{F_{a}}$
where $k$ и $c$ are the empirical coefficients of the equation approximating the second section of the curve of the blade support surface. For blades sharpened with abrasive and el'bor wheels $k=-(0,4-0,5) ; \quad c=5-9$.

The working height can be determined by the formula: $h=R_{\max }(1-\operatorname{sint})$; где: $t=\operatorname{arctg} \frac{u_{1}}{u_{2}}$.
The contact diameter of a single microroughness is found from the expression:
$d=2 R_{\text {max }}$ cost .
Because of mathematical transformations, we obtain, respectively, for each of the stages of the knife:
$Q_{1}=\frac{1-\sin \tau}{\cos \tau} \cdot \frac{b \varepsilon^{v+1}}{v+1} \cdot \frac{F_{\phi}}{F_{a}}$
$Q_{2}=\frac{1-\sin \tau}{\cos \tau} \cdot[k(1-\varepsilon)+c] \cdot \frac{F_{\phi}}{F_{a}}$
It should be noted that formulas (8) and (9) have the same structure: the first factor determines the sliding cutting mode, the second determines the stage of the knife operation (bedding or normal wear), and the third determines the contact conditions of the cutting tool and material.
To determine the actual contact area, let us first consider a single microroughness modeled as a truncated halfcylinder (Fig. 1.a). The actual area can be presented as a sum:
$F_{\phi}=S_{\text {бок }}+S_{\text {сеч, }} ;$
$S_{\text {бок }}$ is the lateral surface of a truncated half-cylinder, which is determined by the formula:
$S_{\text {бок }}=\frac{R_{\max }^{2}}{2}\left[\frac{\pi \cdot \tau}{180}-\sin \tau\right]$
$S_{\text {ceu }}$ is a cylindrical surface in contact with the material. It should be borne in mind that with sliding cutting this value is 2 times less than with cutting.
$S_{c e 4}=4 K_{\max } \cdot \operatorname{tg} \frac{\beta}{2}\left[\left(\frac{a}{2 \operatorname{tg} \frac{\beta}{2}}+R_{\max }\right) \cos \frac{\tau}{2}-R_{\max } \cdot \sin \frac{\tau}{2}\right]$

Then: ${ }_{F_{\phi}}=4 R_{\max } \cdot \operatorname{tg} \frac{\beta}{2}\left[\left(\frac{a}{2 \operatorname{tg} \frac{\beta}{2}}+R_{\max }\right) \cos \frac{\tau}{2}-R_{\max } \cdot \sin \frac{\tau}{2}\right]+R_{\max }^{2}\left(\frac{\pi \cdot \tau}{180}-\sin \tau\right)$
Since $l_{b} / S_{m}$ microroughnesses are involved in the cutting process, the actual contact area is determined by the formula:
$F_{\phi}=\frac{l_{b}}{m}\left\{4 R_{\max } \cdot \operatorname{tg} \frac{\beta}{2}\left[\left(\frac{a}{2 \operatorname{tg} \frac{\beta}{2}}+R_{\max }\right) \cdot \cos \frac{\tau}{2}-R_{\max } \cdot \sin \frac{\tau}{2}\right]+R_{\max }^{2}\left(\frac{\pi \cdot \tau}{180}-\sin \tau\right)\right\}$
The contour contact area is determined by the formula: $F_{a}=a \cdot l_{b}$ where: $a$ is the width of the cutting edge; $l_{b}$ is the base length.
After substituting the expressions for $F_{\phi}$ and $F_{a}$ in the formulas for determining the cutting ability, we get for:

1. stage of bedding:
$Q_{1}=\frac{1-\sin \tau}{\cos \tau} \cdot \frac{b \varepsilon^{v+1}}{v+1} \cdot \frac{1}{S_{m} \cdot a} \cdot \cdot\left\{4 R_{\max } \cdot \operatorname{tg} \frac{\beta}{2}\left[\left(\frac{a}{2 \operatorname{tg} \frac{\beta}{2}}+R_{\max }\right) \cdot \cos \frac{\tau}{2}-R_{\max } \cdot \sin \frac{\tau}{2}\right]+R_{\max }^{2}\left(\frac{\pi \cdot \tau}{180}-\sin \tau\right)\right\}$
2. stage of normal wear:
$Q_{2}=\frac{1-\sin \tau}{\cos \tau} \cdot[k(1-\varepsilon)+c] \cdot \frac{1}{S_{m} \cdot a} \cdot \cdot\left\{4 R_{\max } \cdot \operatorname{tg} \frac{\beta}{2}\left[\left(\frac{a}{2 t g} \frac{\beta}{2}+R_{\max }\right) \cdot \cos \frac{\tau}{2}-R_{\max } \cdot \sin \frac{\tau}{2}\right]+R_{\max }^{2}\left(\frac{\pi \cdot \tau}{180}-\sin \tau\right)\right\}$
Analysis of these dependences causes certain difficulties due to the significant number of parameters included in them, many of which can take on numerical values in a wide range, depending on a number of other factors.

Cutting ability of a knife depends on its geometry and parameters of the microgeometry of the cutting edge, as well as sliding cutting modes. Calculations carried out using experimental data on the kinetics of changes in the microgeometry of the blades over the period of durability show that at the stage of bedding, the cutting ability of the knives, despite the large values of $R_{\max }$, is inferior to the value of $Q_{2}$. This is most likely due to the large values of the second factor, i.e. a large number of micro-teeth involved in the destruction of the material, as well as a decrease in their longitudinal pitch.
It follows from the formulas that an increase in the angle $\tau$ indicating an increase in the sliding motion of the blade, significantly reduces the cutting ability of the knives, both at the stage of bedding and normal wear. Thus, the cutting ability $Q$ is a criterion for the microrelief of the blade and includes the parameters of its longitudinal and transverse microgeometry, as well as the kinematic characteristics of sliding cutting.
Taking into account the real cutting modes of existing machines and devices with lamellar knives [5, 6], the value of $\tau$ was taken equal to $84^{0}$, which corresponds to the slip coefficient $K c=u_{1} / u_{2}=10$, and the value $\varepsilon=$ 0.3 , the value of the knife edge angle is $16^{\circ}$. Numerical the values of the remaining parameters $a, R_{\max }, S_{m}, b$,

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$v, k, c$ were taken from the results of measurements of the microgeometry of sharpened knives that had worked for a certain time.

The range of their change was (in microns): $4.6<\mathrm{a}<31.3 ; 79.1<S m<721.0 ; 12.4<R_{\max }<29.0$. Since the coefficients of the reference curve vary within relatively small limits, to simplify the calculations, we have taken the following constant values: $b=v=2,0 ; k=-0,5 ; c=5,0$.
Calculations have shown that the maximum cutting capacity should be expected from knives with the minimum parameters $a, S_{\max }$ and increased characteristics $R_{\max }$. To check this position, an experimental determination of the cutting ability of thin plate knives was carried out by the method $R_{2}=$ const. The noted significant influence of the parameters $R_{\max }$ and $S_{m}$ on the values of $Q_{1}$ and $Q_{2}$ suggests that modern methods of forming the cutting edge of thin plate knives (finishing the blades, multi-pass grinding, using el`bor abrasive wheels) provide acceptable values of the indicators of the transverse microrelief, but do not fully contribute to obtaining the necessary characteristics of the longitudinal microgeometry of the blade, namely: increased values of $R_{\max }$ and reduced values of the longitudinal step $S_{m}$ of the micro-teeth. When determining the rational values of $R_{\max }$ and $S_{m}$, it should be proceeded from such an important characteristic of the sliding cutting process as the working height of the micro-teeth [7], which takes into account the kinematic parameters of the cutting machine, the shape of the blade of the lamellar knife, and the degree of loading of the cutting edge. Consider the operation of two adjacent teeth (or micro-teeth) 1 and 2 of the blade of a lamellar knife moving at a speed $u_{l}$ (Fig. 2). Material feed rate is $u_{2}$. If we take a point belonging to the material at the top of tooth 1 and use the reference system associated with the knife, we can assume that point 1 moves along the resultant, the components of which are proportional to the mutually perpendicular velocities $u_{1}^{o \sigma p}$ and $u_{2}$, where $u_{1}^{o \sigma p}$ is the inverse speed, equal in magnitude to the speed of the knife $u_{1}$. The meeting of this point with the blade of the next tooth will occur at point 1 ", located at a distance $b$ vertically from the top of the tooth. Line $1-1$ " is the border of the material cut by tooth 1 . Therefore, the segment $2-1^{\prime \prime}$, designated $l_{a}$, will be the working length of the tooth blade, and $p_{p}$ is the working height of the tooth. In Fig. 2 it can be seen that the projection of the movement of point 1 on the vertical t' $S_{m}-b$, where $b=a_{p} \cdot \operatorname{tg} \frac{\theta}{2} ; \theta$ is the contour angle. Thereby
$t^{\prime}=S_{m}-a_{p} \cdot \operatorname{tg} \frac{\theta}{2}$


Fig. 2. Determination of the working height of the micro-teeth of the blade

The duration of time during which point 1 moves horizontally by $1-1$ is $\tau_{0}=\frac{a_{p}}{u_{2}}$; moving by 1-1" vertically will also occur in time $\tau_{0}$ )

After simple transformations we obtain:

$$
\begin{equation*}
a_{p}=\frac{S_{m} \cdot u_{2}}{u_{1}+u_{2} \cdot \operatorname{tg} \frac{\theta}{2}} \tag{16}
\end{equation*}
$$

And further if $u_{1} / u_{2}=\operatorname{tg} \tau=k_{c}$ we finally obtain

$$
\begin{equation*}
a_{p}=\frac{S_{m}}{k_{c}+\operatorname{tg} \frac{\theta}{2}} \tag{17}
\end{equation*}
$$

Hence, it is obvious that the value of $R_{\text {max }}$ should be chosen taking into account the value of $a_{p}$ at fixed values of $Q$ and $S_{m}$. At $R_{\max }>a_{p}$ the bluer blade depressions will not participate in the formation of a new surface, which is desirable from the point of view of improving the cut quality.

Determination of the working height $a_{p}$ of the teeth by formulas (16) and (17) does not cause difficulties at $u_{1} \neq 0 ; u_{2} \neq 0$. In this case, the knife speed $u_{1}$ is a variable depending on the crank angle:
$u_{1}=\omega \cdot r\left(\sin \alpha+\frac{\lambda}{2} \sin 2 \alpha\right)$
where: $\omega$ is angular speed of the crank; $r$ is the radius of the crank; $\alpha$ current crank angle; $l$ is connecting rod
length $\lambda=\frac{r}{l}$;
If by formula (18) calculate $u_{1}$ for different angles of rotation of the crank, then you can get the working height of the teeth as a function of $\alpha$. A more accurate determination of the working height of the tooth with the reciprocating movement of the knife is possible when analyzing the trajectories of the adjacent points of the blade in the material, which gives an expression for the average value of the working height of the tooth in the form:

$$
\begin{equation*}
a_{p}^{c p}=\frac{S_{m} \cdot \pi}{K_{c}^{\max }} \tag{19}
\end{equation*}
$$

Where $K_{c}^{\max }$ is the maximum value of the slip coefficient $K_{c}^{m a c}=\frac{\omega r}{u_{2}}$;
Where: $\omega$ is angular speed of the crank.

## Conclusions

The obtained ratios for calculating the working height of the teeth of lamellar knife are valid both for a serrated blade and for knives with a smooth blade, since at certain cutting conditions, characterized by high $K_{2}$ values, only the vertexes of the micro-teeth will be in operation.

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