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Expansion of Harmonic Wave In the Wedge with Random Vertical Angle

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Abstract: The article covers the expansion of harmonic waves in the deformable wedge. A spectral problem is formulated, which is solved numerically. The article reports the development of the method for solving the problem, the action of elastic wave on cylinder with a radial crack by limiting case of wedge with angle of 360° on bodies (shell) located in infinite linear-elastic medium, as well as its algorithms. Closed system of differential equations, as well as the corresponding initial and boundary conditions have been drawn. Obtained analytical results have theoretical and applied significance.

Introduction. The article reports the development of the method for solving the problem, the action of elastic wave on cylinder with a radial crack by limiting case of wedge with angle of 360° on bodies (shell) located in infinite linear-elastic medium. Elastic cylinder with a radial crack is the limiting case of wedge with an angle of 360° [1].

Three groups of relations specify the main equations of motion of elastic medium occupying region B

$$div\sigma = \rho \frac{\partial^2 \vec{u}}{\partial t^2} \qquad (1)$$

$$\varepsilon = \frac{1}{2}\left[grad\vec{u} + \left(gard\vec{u}\right)\right] \qquad (2)$$

$$\tau = \lambda div\vec{u} \cdot E + 2\mu\varepsilon \qquad (3)$$

The tilde denotes the operation of transposition of square matrix; $\hat{E}$ is unit tensor of the second rank; elastic moduli, called Lame constants; λ and μ are complex value. If $\eta = 0$, then λ and μ are real numbers (Lame constants) [1, 2, 8]. In a cylindrical coordinate system, equations (1), (2), (3) have the form

$$\rho \frac{\partial u_r}{dt^2} = \frac{\partial \sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{r\varphi}}{r} + \frac{1}{r}\frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{\partial \sigma_{rz}}{\partial z};$$

$$\rho \frac{\partial^2 u_\varphi}{\partial t^2} = \frac{1}{r}\frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{2\sigma_{r\varphi}}{r} + \frac{\partial \sigma_{r\varphi}}{\partial r} + \frac{\partial \sigma_{z\varphi}}{\partial z}; \qquad (4)$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{zz}}{r} + \frac{1}{r}\frac{\partial \sigma_{z\varphi}}{\partial \varphi};$$

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$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}; \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}; \quad \varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{u_r}{r}; \qquad (5)$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right); \ \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right);$$

$$\varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right); \ \sigma_{rr} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_r}{\partial r};$$

$$\sigma_{r\varphi} = 2\mu\varepsilon_{r\varphi} = \mu\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right);$$

$$\sigma_{rz} = 2\mu\varepsilon_{rz} = \mu\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right); \qquad (6)$$

$$\sigma_{\varphi\varphi} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r}\right);$$

$$\sigma_{\varphi z} = \mu\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right);$$

$$\sigma_{zz} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_z}{\partial z}.$$

Where $\sigma_{rr}, \sigma_{r\varphi}, \sigma_{rz}, \sigma_{\varphi\varphi}, \sigma_{\varphi z}, \sigma_{zz}$ are the stress tensor components respectively; $\varepsilon_{rr}, \varepsilon_{r\varphi}, \varepsilon_{rz}, \varepsilon_{\varphi\varphi}, \varepsilon_{\varphi z}, \varepsilon_{zz}$ are the components of the strain tensor respectively. The relationship between stresses and strains is given in the second chapter (6). Relations (4), (5), (6) after identical algebraic transformations are reduced to a system of six differential equations solved with respect to the first derivative with respect to the radial coordinate

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$$\begin{cases} \dfrac{\partial u_r}{\partial r} = \dfrac{1}{K}\sigma_{rr} - \dfrac{\lambda}{K}\left(\dfrac{1}{r}\dfrac{\partial u_\varphi}{\partial \varphi} + \dfrac{u_r}{r} + \dfrac{\partial u_z}{\partial z}\right); \\ \dfrac{\partial u_\varphi}{\partial r} = \dfrac{1}{\mu}\sigma_{r\varphi} - \dfrac{1}{r}\left(\dfrac{\partial u_r}{\partial \varphi} - u_\varphi\right); \\ \dfrac{\partial u_z}{\partial r} = \dfrac{1}{\mu}\sigma_{rz} - \dfrac{\partial u_r}{\partial z}; \\ \dfrac{\partial \sigma_{rr}}{\partial r} = \rho\dfrac{\partial^2 u_r}{\partial t^2} - \dfrac{\tilde{A}}{r} - \dfrac{1}{r}\dfrac{\partial \sigma_{r\varphi}}{\partial \varphi} - \dfrac{\partial \sigma_{rz}}{\partial z}; \\ \dfrac{\partial \sigma_{r\varphi}}{\partial r} = \rho\dfrac{\partial^2 u_\varphi}{\partial t^2} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\left[\sigma_{rr} - \tilde{A}\right] - \dfrac{2\sigma_{r\varphi}}{r} - \dfrac{\partial}{\partial z}\tilde{B}; \\ \dfrac{\partial \sigma_{rz}}{\partial r} = \rho\dfrac{\partial^2 u_z}{\partial t^2} - \dfrac{\partial}{\partial z}\left[\sigma_{rr} - 2\mu\left(\dfrac{\partial u_r}{\partial r} - \dfrac{\partial u_z}{\partial z}\right)\right] - \dfrac{\sigma_{rz}}{r} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\tilde{B}; \end{cases} \quad (7)$$

where the following notations are introduced

$$\tilde{A} = 2\mu\left[\frac{\partial u_r}{\partial r} - \frac{1}{r}\left(\frac{\partial u_\varphi}{\partial \varphi} + u_r\right)\right]; \quad \tilde{B} = \mu\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right).$$

boundary conditions set in the form:

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}; \qquad \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0 \quad (8)$$

Periodicity conditions allows eliminating the dependence of the main unknowns on time and the axial coordinate z using the following change of variables:

$$\begin{aligned} \sigma_{rr} &= \sigma(r,\varphi)\cos(kz - \omega t); \\ u_r &= w(r,\varphi)\cos(kz - \omega t); \\ u_\varphi &= v(r,\varphi)\cos(kz - \omega t); \\ \sigma_{rz} &= \tau_z(r,\varphi)\sin(kz - \omega t). \\ \sigma_{r\varphi} &= \tau_\varphi(r,\varphi)\cos(kz - \omega t); \end{aligned} \quad (9)$$

Under condition (8), separation of the variables r and φ is impossible. Considering (9), the system of equations (7) takes the form:

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$$\begin{cases} w' = \dfrac{\sigma}{k} - \dfrac{\lambda}{k}\left(ku + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right) \\ v' = \dfrac{\tau_\varphi}{\mu} + \dfrac{1}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right) \\ u' = \dfrac{\tau_z}{\mu} + kw \\ \sigma' = -\omega^2 \rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_\varphi}{\partial \varphi}\right) - k\tau_z \\ \tau'_\varphi = -\omega^2 \rho v - \dfrac{1}{r}\left(\dfrac{\partial (A+\sigma)}{\partial \varphi} + 2\tau_\varphi\right) - kB \\ \tau'_z = -\omega^2 \rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + k(\sigma + 2\mu(ku - w')) \end{cases} \quad (10)$$

where $A = 2\mu\left(\dfrac{1}{2}\left(\dfrac{\partial v}{\partial \varphi} + w\right) - w'\right)$ $B = \mu\left(\dfrac{1}{r}\dfrac{\partial u}{\partial \varphi} - kv\right)$

The boundary conditions are transformed similarly (8)

$$r = 0, R: \quad \sigma = \tau_\varphi = \tau_z = 0 \;. \quad (11)$$

It is easy to see that the components of the stress tensor $\sigma_{\varphi\varphi}$, $\sigma_{\varphi z}$ and σ_{zz} are expressed in terms of the main unknowns by the formulas:

$$\sigma_{\varphi\varphi} = \sigma_{rr} + 2\mu\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} - \frac{\partial u_r}{\partial r}\right), \;\; \sigma_{\varphi z} = \mu\left(\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_\varphi}{\partial z}\right) \quad (12)$$

$$\sigma_{zz} = \sigma_{rr} + 2\mu\left(\frac{\partial u_z}{\partial z} - \frac{\partial u_r}{\partial r}\right)$$

Then, considering the first equation of system (12), boundary conditions (11) take the form:

$$\sigma_\varphi = A + \sigma_r = a\sigma_r + b\frac{1}{r}\left(\frac{\partial v}{\partial \varphi} + w\right) + cku = 0$$

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}; \qquad \tau_\varphi = 0 \quad (13)$$

$$B = \mu\left(\frac{\partial u}{r\partial \varphi} - kr\right) = 0$$

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where

$$a = 1 + \frac{2\mu}{k} \quad b = 2\mu\left(1 + \frac{\lambda}{k}\right) \quad c = 2\mu\frac{\lambda}{k}$$

Boundary-value problem for the system of equations in frequency derivatives (13) is solved by differential equations using the method of straight lines, which will allow using the software apparatus of the orthogonal marching method in the solution. According to the method of straight lines, the rectangular domain of definition of the function of the main unknowns is covered by straight lines parallel to the *r* axis and evenly spaced from each other (Fig. 1) [5,6,11].

The solution is sought only on these straight lines, and the derivative in the direction φ is replaced by approximate finite differences. The second-order approximating formulas used for the first and second derivatives have the form:

$$y_{i,\varphi} \cong \frac{y_{i+1} - y_{i-1}}{2\Delta} \cong \frac{-3y_i + 4y_{i+1} - y_{i+2}}{2\Delta} \cong \frac{3y_i - 4y_{i-1} + y_{i-2}}{2\Delta} \quad (14)$$

$$y''_{i,\varphi} \cong \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta^2} \quad (15)$$

where *i* changes from *0* to $N+1\left(i = \overline{0, N+1}\right), y_i$ is the projection of unknown function onto the line with the number *i*; Δ is step of partition along the coordinate φ

Because of discretization, the vector of main unknowns of total dimension *6N* can be written as:

$$Y = \left(\{w_i\}, \{v_i\}, \{u_i\}, \{\sigma_{ri}\}, \{\tau_{\varphi i}\}, \{\tau_{zi}\}\right)^T \quad i = \overline{1, N} \quad (16)$$

Therefore, the initial spectral problem (10), (11) with the help of discretization of the coordinate φ by the method of lines is reduced to the canonical problem (16), for the solution of which the used method of orthogonal marching is applied. Table I shows the limiting values of the phase velocity of the first edge mode depending on the angle of the wedge at the apex (in terms of the thickness of the wedge at the base h_2) (column 1), found for the material with Poisson coefficient $\upsilon = 0.25$ according to the theory of Kirchhoff-Love plates (column 2) [8, 13, 19], Timoshenko (column 3), in frame of the method for calculating a three-dimensional wedge set out in this article (columns 4-5) and by the formula $C_0 = C_ч \sin(m\varphi)$ [12,16,19], m = 1, 2, …, $m\varphi < 90°$ (column 6). Column 4 corresponds to the design option with three internal straight lines (N = 3) and boundary conditions (8), column 5 corresponds to the boundary conditions:

$$\varphi = -\frac{\varphi_0}{2}; \sigma_{\varphi\varphi} = \sigma_{\varphi ч} = \sigma_{\varphi z} = 0; \varphi = 0; u_r = u_z = \sigma_{\varphi\varphi} = 0$$

with the same number of lines. In accordance with the numerical results, and given in Table 1, the calculation options using the Kirchhoff - Love, Timoshenko and three-dimensional theory methods agree with each other within 7% for wedge angles with a base thickness h_2 not exceeding 0.5 (*wedge angle* $\varphi_0 = 28$ °).

Thereby, as opposed to waveguides with rectangular cross-section in wedge-shaped waveguides with sufficiently small wedge angle, it is permissible to use the theory of Kirchhoff - Love plates in the analysis of the dispersion dependences of the first mode. The established fact is explained by the phenomenon of

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localization of the vibration mode near the acute angle of the wedge, described in [12]. This phenomenon should be considered as a characteristic feature of the dynamic behavior of a plate of variable thickness.

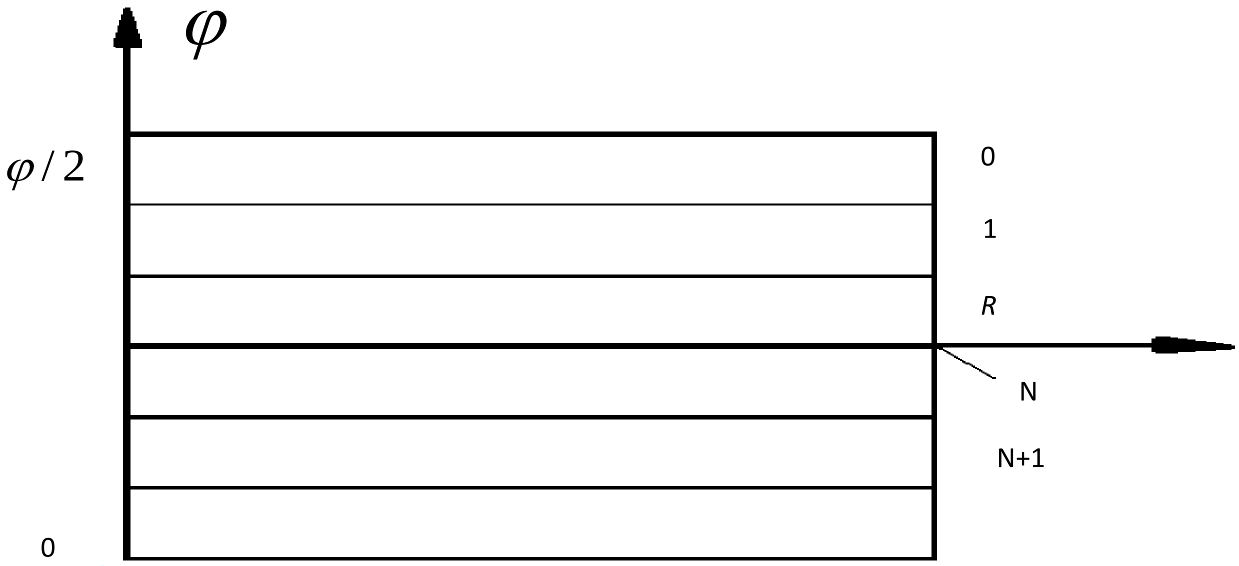

Fig. 1. Design diagram

Based on the results obtained, the following conclusions were drawn.:

results of calculating the limiting velocity of expansion of the first mode in a wedge-shaped waveguide according to the theory of Kirchhoff - Love plates and according to the dynamic theory of elasticity differ by no more than 6% for the angles of the wedge apex not exceeding $28^0 < \varphi < 90^0$. As a result, the calculations differ by up to 20%. [12, 18, 19]

Table 1

I_2	φ_0	*K/A*	*T*	*3(1)*	*3(2)*	*A*
0.2	11^0	0.2	0.196	-	-	0.182
0.3	17^0	0.3	0.286	0.308	0.298	0.276
0.5	28^0	0.5	0.442	0.475	0.462	0.433
0.7	38^0	0.7	0.563	0.605	0.592	0.574
1	53^0	1	0.691	0.741	0.729	0.736
2	90^0	2	0.864	0.908	-	0.92

References

1. Guz A.N., Kubenko V.D., Cherevko M.A. Diffraction of elastic waves. Kiev: Naukova Dumka, 1978, p. 308.
2. Guz A.N., Golovchan V.T. Diffraction of elastic waves in multiply connected bodies. Kiev: Naukova Dumka, 1972, p. 254.
3. I.I. Safarov, Z.F. Djumaev, Z.I. Boltaev. Harmonic waves in infinite cylinder with radial crack considering the damping capacity of the material. Problems of mechanics. 2011, p. 20-25.

https://journals.researchparks.org/index.php/IJHCS e-ISSN: 2615-8159 | p-ISSN: 2615-1898
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4. I.I. Safarov, M.Kh. Teshaev, Z.I. Boltaev. Mathematical modeling of wave process in mechanical waveguide considering internal friction. Germany. LAP. 2013, p.243.
5. Mamatkulov Sh. Oscillations and waves in elastic and soil media. Tashkent: Fan, 1987, p.104.
6. Mirsaidov M.M., Troyanovskiy I.E. Dynamics of inhomogeneous systems considering internal dissipation and wave entrainment of energy. Tashkent: Fan, 1990, p. 207
7. Muborakov Ya.N., Safarov I.I., Sobirov M.I., Atoev A.B. About the main methods of studying the stress-strain state of underground cylindrical structures when interacting with elastic waves. Collection of papers “Strength of engineering structures under seismic and impulsive influences.” Tashkent: Fan. 1990, p. 67-83.
8. Rashidov T.R. Dynamic theory of seismic resistance of complex systems of underground structures. Tashkent: Fan, 1973, p. 180-182
9. Gaybullaev Z.Kh., Boltaev M.B., Kenjaeva M. About natural vibrations of body in elastic medium // Modern problems in machine mechanics. Collection of reports of republic conference. Tashkent, 2004 p. 251-252.
10. Newmark N.M., A. Method of Computation for Structural Dynamics. - J. Eng. Mech. Div. Proc. ASCE, 1959, vol. 85, no.7, p. 67 - 94.
11. Terzagni K, Theoretical Soil Mechanics, Wily, New Yok, 1943, p. 194.
12. Me Nelty Y.W. An Experimental study of Arching in Sand, US Waterways Experiment Station, Rep. 1-674.
13. Gaybullaev, Z.Kh., Juraev, T.Kh., Jumaev, Z. “On expansion of elastic waves near the edge of a strip of variable thickness.” Republican scientific conference on mechanics dedicated to the 90th anniversary of academician M.T. Urazbayev. Tashkent, 1996, p. 231-232.
14. Razzokov Sh.I., Temirov S.T., Jumaev Z.F. Wave expansion into a layer located in infinite elastic medium. // 2500th anniversary of Bukhara city. Bukhara, 1997, p. 182-183.
15. Safarov I.I., Yadgarov U., Juraev T.O., Jumaev Z.F. On steady vibrations of three-layer cylindrical bodies. // Problems of mechanics, Vol. 1. Tashkent, 2000, p. 31-34
16. Safarov I.I., Razzokov Sh.I., Jumaev Z.F. Oscillations of elastic cylindrical body under the action of internal pressure. DAN 2000, Vol. 11, p. 25-27.
17. Safarov I.I., Avliyokulov N.N., Jumaev Z.F. “Oscillations of body mounted on viscoelastic supports.” Dedicated to the 60th anniversary of Z.Sh. Jumaev and T.Kh.Sharipov. Bukhara State University. Bukhara, 2001, p. 434-437.
18. Safarov I.I., Aslonov B., Jumaev Z.F. “On natural vibrations of dissipative inhomogeneous viscoelastic mechanical systems”. Modern problems of algorithmization and programming. Tashkent, 2001, p. 127-128.
19. Love A. Mathematical theory of elasticity. Moscow: Gostekhizdat, 1935, p. 674.