

# Methodology for the Design and Establishment of Synthetic Fish Pond Bio Systems and Fish Harvest Optimization Planning

*Odinazoda S. A., Nimatova M. Kh.*

Tajik National University

*B. Elmurodova*

Associate professor of information technology department of Karshi branch of TUIT named after Muhammad Al-Kharazmi

-----\*\*\*-----

## 1. Design and creation of a conceptual and mathematical model of a productive fish pond ecosystem

We show that the intensity of microbiological activities in a complex biodynamics of the entire aquatic system is the basis for high fish production, based on the biological basis of mathematical modeling of productive fish pond ecosystems.

The investigation of biological production processes in Tajikistan reservoirs revealed that they are mostly of the olithographic kind. The majority of them are in rivers that have been flooded by snow and ice. Fish ponds are the most practical facilities for management, which must obtain high-quality goods while lowering production costs, among freshwater bodies of water.

Due to the employment of complicated management elements, the intensity of matter and energy transformation in a very productive ecosystem, fish ponds, and the simplicity of its trophic structure permits high efficiency of aquatic species production (supply of organic and mineral fertilizers, feed).

Obtaining a precise mathematical model of water's biological processes requires not only a thorough understanding of its life, but also the capacity to recognize the most significant of them and predict how they will grow in the reservoir.

To catch fish, a mathematical model of the fish pond ecosystem requires extensive knowledge of all levels of the bacteria and phytoplankton ecosystem. Microbiological processes are the most ambitious in comparison to the subsequent links in the food chain, because the total energy contained in the bodies of the two trophic levels (phytoplankton and bacterioplankton) determines the biological productivity of the water bodies' subsequent links in the trophic chain, including fish products. Some mathematical modeling of ecosystems such as reservoirs and fish ponds hasn't given enough consideration to bacterial removal.

As a result of the decomposition of organic matter, water is saturated with carbon dioxide, and complex organic compounds of nitrogen, phosphorus, and numerous other elements are released, which are then reoccupied with the development of microbial biomass. While dissolved carbon dioxide in water is the most convenient form, it is mostly utilized in phytoplankton photosynthesis. We consider photosynthesis and bacterial degradation to be the only microbiological processes associated to phytoplankton in this study.

The circulation of substances in this process is depicted in the diagram below.

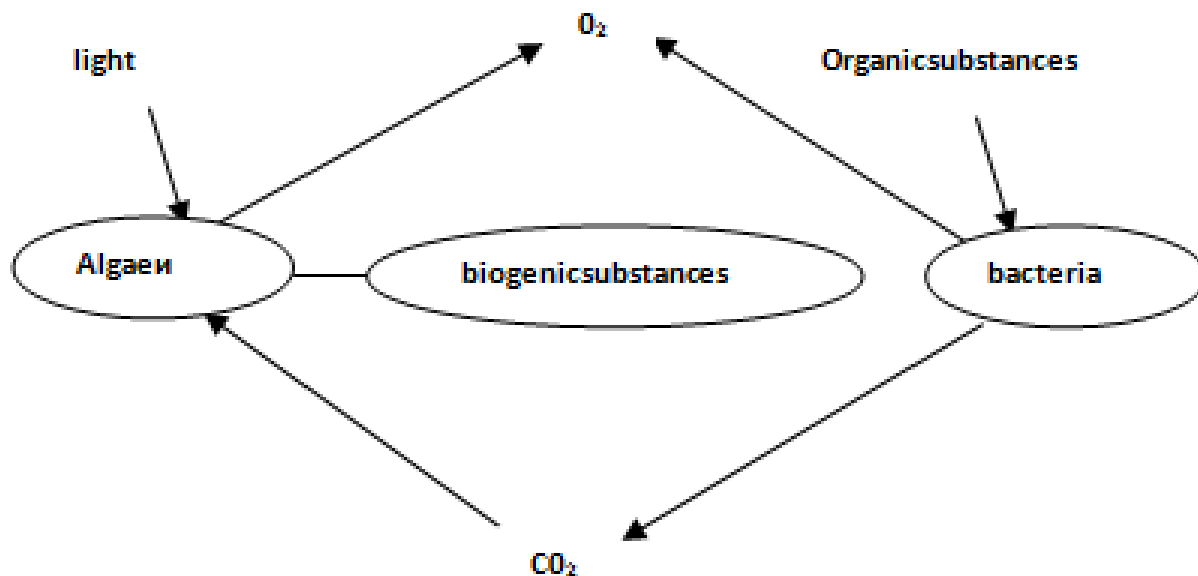


Fig.1. Schematic diagram of the circulation of substances

The effects of fertilizers on the microorganisms that carry out the nutrient water cycle have been thoroughly investigated. It is well recognized that high-end production in fish ponds is nearly impossible without the creation of nutrients.

To ensure the rapid development of microbiological processes, precise nitrogen and phosphorus water concentrations must be maintained at all times. The correct nitrogen-to-phosphorus ratio helps maintain a particular percentage of reservoir plankton (40-50 percent), which is the most desirable feed. Biotechnology is also thought to have created organic particles in the fish pond. Cattle fertilizer is frequently utilized for this purpose in the practice of water body fish. Organic matter considerably improves microbiological processes, but they can only have a good effect if enough water is saturated with nutrients. The biological activities taking place in the gas fish ponds of the regime are closely related. Photosynthesis is the primary cause of oxygen saturation. The addition of carbon dioxide to water as a result of aquatic organisms' respiration, with bacteria playing a key role. High rates of photosynthetic processes, on the other hand, should result in bacterial degradation of organic matter and carbon dioxide saturation of the masses' water.

The mix of fish types can be controlled to promote maximum fish production. White amur and braggart, white amur, and buffalo are commonly grown in carp polyculture. This relationship is thought to print the best possible use of natural fodder. Silver carp are phytoplankton, Bragg carp are zooplankton, buffalo and carp are benthos, and white amur are macrophytes.

The image depicts a model scheme that incorporates all of the aforementioned reasons for the biological basis of mathematical modeling of highly productive fish pond ecosystems.

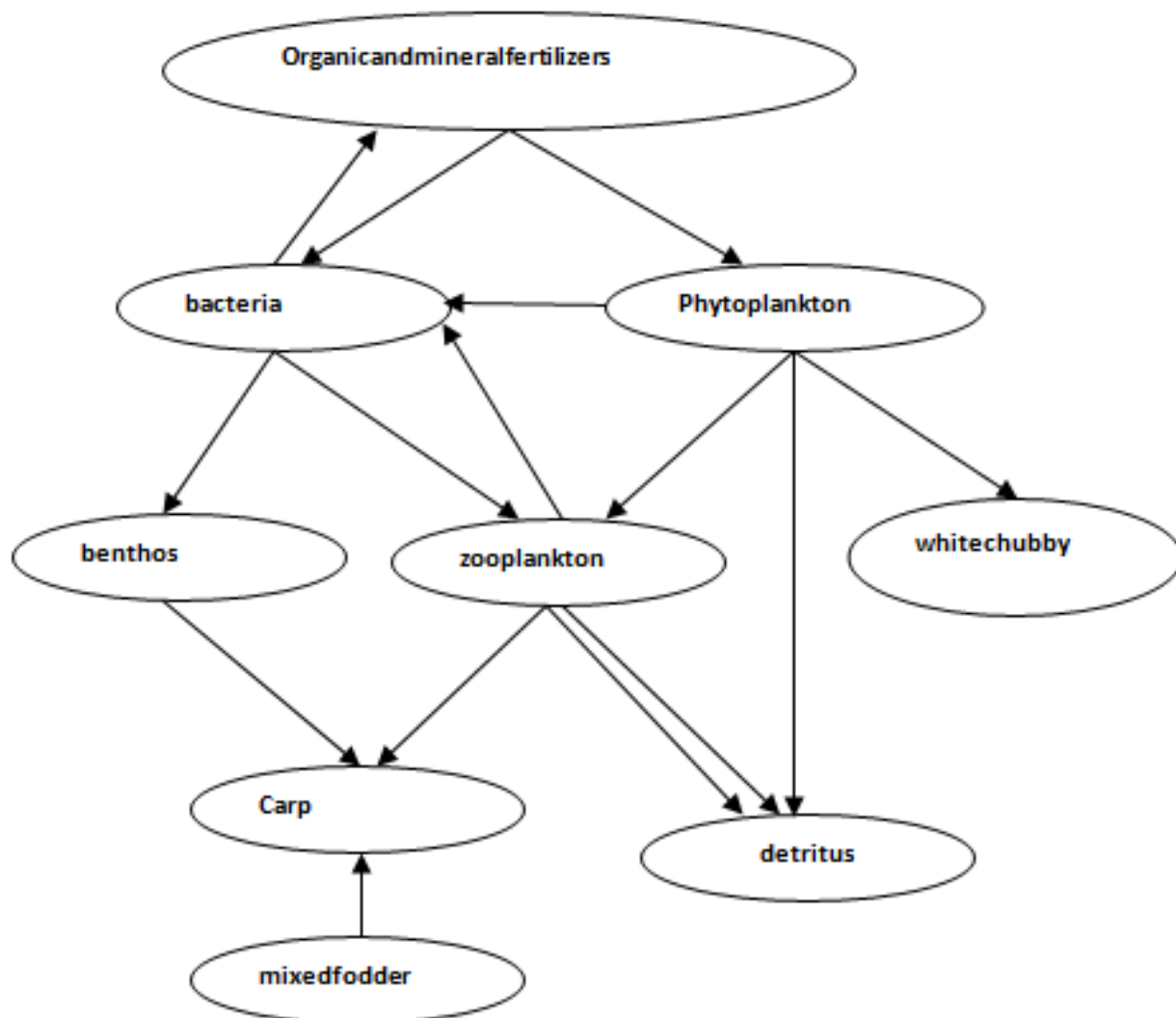


Fig.2. Conceptual model of biological bases

In comparison to previous models, we allocated bacterial activity a proper role. Entering the reservoir of organic and mineral fertilizers allows for very productive ecosystem management and lending. Furthermore, the biological demand for fertilizers is a prerequisite for their application in fish cages. Only two species of carp fish and silver carp were added into the model's design. Other species (carp, white amur, and buffalo) have a little impact on the reservoir's ecological processes. Only if biological processes in all regions of the Baltic Chain are at a high level can the plan meet the requirements of highly productive ecosystems.

This scheme can lead to another form as follows [1]

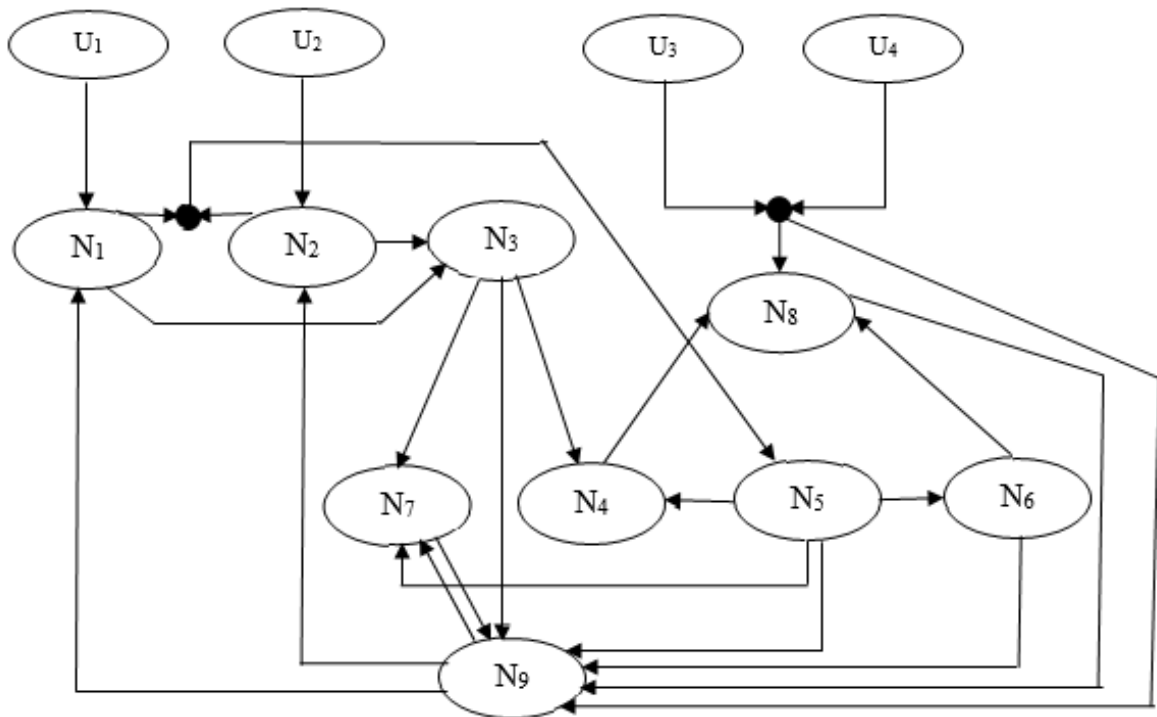


Fig. 3. A conceptual model of the productive ecosystems of a fish pond.

Here N8-, N7-silver carp, N3-phytoplankton, N6-benthos, N4-zooplankton, N5-bacteria, N1-soluble inorganic phosphorus, nitrogen N2-rastvorimy neorganichönny, N9-shards. So we have nine state parameters for the model. The input functions are climatic factors - water temperature (T) and solar radiation intensity (I0) on the surface of the reservoir. Also included are four control functions characterizing the introduction of artificial feed (U3-feed, U4-silkworm pupae) and mineral fertilizers (U1-superphosphate, U2-ammonium nitrate).

The main goal of this research is to analyze the model to create a general methodology for designing and creating very persistent synthetic biosystems that plan for optimal fish pond operation to maximize yields.

The mathematical model of the productive ecosystem of the fish pond was carried out on the computer.

Here we obtain a system of ordinary distinctive equations, which is solved by Runge-Kutta-Merson.

The resulting system is as follows.

$$\begin{aligned}
 \frac{dN_1}{dt} &= a_{91}N_9N_1 - a_{13}N_1N_3 - a_{15}N_1N_5 + U_1(t), \\
 \frac{dN_2}{dt} &= a_{92}N_9N_2 - a_{23}N_2N_3 - a_{25}N_2N_5 + U_2(t), \\
 \frac{dN_3}{dt} &= a_{13}N_1N_3 + a_{23}N_2N_3 + a_{cu3}N_{cu}N_3 - a_{37}N_3N_7 - a_{34}N_3N_4 - a_{39}N_3N_9, \\
 \frac{dN_4}{dt} &= a_{34}N_3N_4 + a_{54}N_5N_4 - a_{48}N_4N_8 - a_{49}N_4N_9, \\
 \frac{dN_5}{dt} &= a_{15}N_1N_5 + a_{25}N_2N_5 + a_{cu5}N_{cu}N_5 - a_{54}N_5N_4 - a_{56}N_5N_6 - a_{57}N_5N_7 - a_{59}N_5N_9, \\
 \frac{dN_6}{dt} &= a_{56}N_5N_6 - a_{68}N_6N_8 - a_{69}N_6N_9, \\
 \frac{dN_7}{dt} &= a_{37}N_3N_7 + a_{57}N_5N_7 - a_{97}N_9N_7 - a_{79}N_7N_9, \\
 \frac{dN_8}{dt} &= a_{68}N_6N_8 + a_{48}N_4N_8 - a_{89}N_8N_9 + (\alpha \cdot U_3(t) + \beta \cdot U_4(t)) \cdot N_8, \\
 \frac{dN_9}{dt} &= a_{39}N_3N_9 + a_{49}N_4N_9 + a_{59}N_5N_9 + a_{69}N_6N_9 + a_{79}N_7N_9 + a_{89}N_8N_9 \\
 &+ a_{91}N_9N_1 - a_{92}N_9N_2 - a_{9cu}N_9N_{cu} + (1 - \alpha) \cdot U_3(t) + (1 - \beta) \cdot U_4(t) - K \cdot N_9.
 \end{aligned} \tag{1}$$

$$A = \begin{pmatrix}
 0 & 0 & -a_{13} & 0 & -a_{15} & 0 & 0 & 0 & 0 \\
 0 & 0 & -a_{23} & 0 & -a_{25} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -a_{34} & 0 & 0 & -a_{37} & 0 & -a_{39} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_{48} & -a_{49} \\
 0 & 0 & 0 & -a_{54} & 0 & -a_{56} & -a_{57} & 0 & -a_{59} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_{68} & -a_{69} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_{79} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_{89} \\
 -a_{91} & -a_{92} & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{pmatrix} \tag{2}$$

$$\frac{dN}{dt} = A(N)N + BU \tag{3}$$

Here,  $\alpha, \beta$  – is the corresponding coefficient of sedimentation and pupa for carp consumption, SED is the coefficient of sedimentation.

This system is a special case of the following system:

$$\frac{dN_i}{dt} = f_i(N_i, u), \quad i = \overline{1,9}, \quad 0 \leq t \leq t_k \quad (4)$$

$$N_0 = N(0), \quad N = N(t), \quad u = u(t)$$

$u = (u_1, u_2, u_3, u_4)$  - function vector.

The further task is to choose the optimal value of the function  $u_1, u_2, u_3, u_4$ , at which the functional

$$I(u) = \int_0^{t_k} f(N_i, u) dt - \max \quad (5)$$

reaches the maximum value. This means that with the optimal presentation of feed we get a high yield of fish.

## 2. Planning the optimization of the fish harvesting process

Consider an isolated fish population averaged by age and spatial parameters, described by the logistic model

$$\begin{cases} \frac{dN}{dt} = \delta(t) N(t) - \varepsilon(t) N^2(t), \\ N(0) = N_0 \end{cases} \quad 0 \leq t \leq t_k, \quad (7)$$

where  $\delta = \delta(t)$  coefficient of natural growth of the fish population,  $\varepsilon = \varepsilon(t)$  - coefficient of intraspecific competition, and  $\varepsilon(t) \geq 0$ ,  $t_k$  - is the end of the season.

To optimize the process of growing the fish population, let's introduce a control parameter in the form of biofeedback according to the law  $u = u(t)$ :

$$u \in U = \left\{ \begin{array}{l} u = u(t); \quad u_{\min} \leq u(t) \leq u_{\max} \\ \kappa.H.y. \quad u_{\min} > 0, \quad 0 \leq t \leq t_k \end{array} \right\}. \quad (8)$$

It is natural to assume that the introduction of biofeedstock increases the growth rate and then equation (7) takes the following form

$$\frac{dN}{dt} = (\delta + u)N - \varepsilon N^2, \quad 0 \leq t \leq t_k.$$

To determine the optimal optimization law for the fish population, consider the following functional(s):

$$1. \quad I(u) = \int_0^{t_k} C_1 N(t) dt + C_1^0 N(t_k) - \max \quad (9)$$

(min)

$$2. \quad I(u) = t_k - \min, \quad (10)$$

$$3. I(u) = \int_0^{t_k} [N(t) - N^*]^2 dt - \min . \tag{11}$$

Consequently, the problem of optimizing the cultivation of the fish population can be written in the following form

$$\begin{cases} \frac{dN}{dt} = [\delta(t) + u(t)]N - \varepsilon(t)N^2 \\ N(0) = N_0, & 0 \leq t \leq t_k \\ I(u) - \min, & u \in U \end{cases} \tag{12}$$

Instead of the first and second equations (12) we will consider the problem

$$\begin{cases} \frac{dN}{dt} = \tilde{u}N - \varepsilon N^2, & 0 < t \leq t_k \\ N(0) = N_0, \end{cases} \tag{13}$$

where  $\tilde{u} = \delta + u$ . Without the restriction of generality, we can assume that  $\delta = 0$ , i.e.  $\tilde{u} \equiv u$ . So we have the equation  $\frac{dN}{dt} = \tilde{u}N - \varepsilon N^2$  subject to  $N(0) = N_0$ .

Introduce the substitution  $N = \frac{1}{M}$ , and then  $\dot{M} = -\frac{1}{M^2} \cdot \dot{N}$ , then divide both parts of the last equation by  $\varepsilon$  and get the equation

$$\dot{M} = -uM + \varepsilon, \quad M(0) = M_0. \tag{14}$$

Thus, we obtain a linear differential equation of order 1 with variable coefficients, and it is easy to see that its solution appears in the form

$$M(t) = M_0 e^{\int_0^t u(\tau) d\tau} + \int_0^t \varepsilon(\tau) e^{\int_\tau^t u(\xi) d\xi} d\tau .$$

Then, taking into account the substitution of a  $N = \frac{1}{M}$  we obtain the formula

$$N(t) = \frac{N_0 e^{\int_0^t u(\tau) d\tau}}{1 + N_0 \int_0^t \varepsilon(\tau) e^{\int_0^\tau u(\xi) d\xi} d\tau} . \tag{15}$$

The resulting function can be graphically characterized as follows ( $\varepsilon(\tau) \rightarrow \varepsilon, u \rightarrow u_0$ )

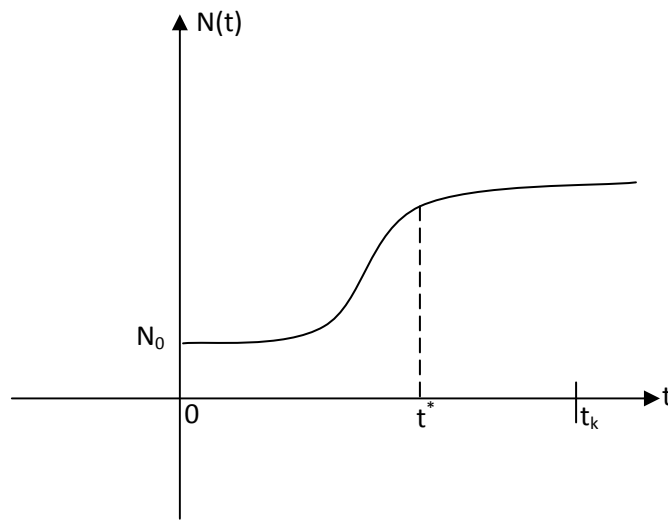


Рис.4. General view of function (15) for arbitrary input data

It follows from Fig. 4 that the control process within the model function (15) should be implemented up to the moment  $t \leq t^*$ , and from the moment  $t > t_k$  it is necessary to keep this control, as the fish biomass remains unchanged. Let's define the time moment  $t^*$ . This moment corresponds to condition  $\frac{dN}{dt} \rightarrow 0$  and therefore  $t^*$

is determined from the formula 
$$N(t^*) = \frac{u(t^*)}{\varepsilon(t^*)}$$

Therefore, taking into account (15) we have

$$u(t^*) = \frac{N_0 \varepsilon(t^*) e^{\int_0^{t^*} u(\tau) d\tau}}{1 + N_0 \int_0^{t^*} \varepsilon(\tau) e^{\int_0^{\tau} u d\xi} d\tau} \tag{16}$$

To solve the optimization problem within the framework of the model (7)

$$\begin{cases} \dot{N} = UN - \varepsilon N^2, & N(0) = N_0 \\ I(u) = \int_0^{t_k} C(t)N(\tau)d\tau, & u \in U \end{cases}$$

Let's form a Hamilton-Pontryagin function

$$H(N, \psi) = -CN + (UN - \varepsilon N^2)\psi$$

And from there we have



$$\dot{\psi} = C - U\psi + 2\varepsilon N\psi, \psi(t_0) = 0,$$

$$\text{i.e. } \psi(t) \equiv -\int_{t_0}^t C e^{+\int_t^\tau (u-2\varepsilon N)d\xi} d\tau < 0.$$

It is easy to see that

$$u^* = \begin{cases} u_{\min}, \psi(t) < 0 \\ u_{\max}, \psi(t) \geq 0 \end{cases}.$$

If we consider the functionality  $I(u) = \int_0^{t_k} c N d\tau + c_0 N(t_k)$ , then  $\psi(t_k) = -c_0$  and therefore

$$\psi(t) = -c_0 e^{\int_0^{t_k} (u-2\varepsilon N)d\xi} - \int_0^{t_k} c e^{\int_t^\tau (u-2\varepsilon N)d\tau} d\tau$$

is  $\psi(t) \leq 0$  for all  $t \in [0, t_k]$ .

**Assertion:** The solution of the problem of cost minimization in fish farming is determined by the parameter  $u^* = u_{\min}$ .

Now consider the problem of maximizing the value of the harvest associated with the model (7). Let  $c = c(\tau)$  the value of one unit of biomass of farmed fish. Then

$$I(u) = \int_0^{t_k} c(t) N(t) dt$$

is total sales revenue  $N$  – number of fish. Consider the problem of maximizing  $I(u)$  on a set of  $U$ . Since

$$I(u) = \int_0^{t_k} c(t) \left[ N_0 e^{\int_0^t u(\tau) d\tau} + N_0 \int_0^t \varepsilon(\tau) e^{\int_\tau^t u(\xi) d\xi} d\tau \right] dt.$$

Let's calculate the gradient of the functional

$$\Delta I = I(u + \Delta u) - I(u) = \int_0^{t_k} \frac{c(t) N_0 e^{\int_0^t (u+\Delta u) d\tau}}{1 + N_0 \int_0^t \varepsilon(\tau) e^{\int_\tau^t (u+\Delta u) d\tau}} - \int_0^{t_k} \frac{c(\tau) N_0 e^{\int_0^\tau u d\tau}}{1 + \int_0^\tau \varepsilon(\tau) e^{\int_\tau^t u d\xi}} =$$

$$\begin{aligned}
 &= \int_0^{t_k} \left[ \frac{c(\tau)N_0 e^{\int_0^t u d\tau} \left( 1 + \int_0^t \Delta u d\tau + \left( \int_0^t \Delta u \right)^2 \right)}{1 + N_0 \int_0^t \varepsilon(\tau) e^{\int_\tau^t u d\xi} \left( 1 + \int_\tau^t \Delta u d\xi + \left( \int_\tau^t \Delta u \right)^2 \right) dt} - \frac{c(t)N_0 e^{\int_0^t u d\tau}}{1 + N_0 \int_0^t \varepsilon(\tau) e^{\int_\tau^t u d\xi} d\tau} \right] dt = \\
 &= \int_0^{t_k} \left[ \frac{c(t)N_0 e^{\int_0^t u d\xi} \left( 1 + \int_0^t \Delta u d\tau + o(\Delta u) \right)}{1 + N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} dt + N_0 \int_0^t \varepsilon(\tau) e^{\int_\tau^t u d\xi} \cdot \int_\tau^t \Delta u d\tau + o(\Delta u)} - \frac{c(t)N_0 e^{\int_0^t u d\tau}}{1 + N_0 \int_0^t \varepsilon(\tau) e^{\int_\tau^t u d\xi} d\tau} \right] dt = \\
 &= \int_0^{t_k} \left[ \frac{c(t)N_0 e^{\int_0^t u d\xi}}{\left( 1 + N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} dt \right)} \cdot \frac{1 + \int_0^t \Delta u d\tau + o(\Delta u)}{1 + \frac{N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} \cdot \int_\tau^t \Delta u d\xi d\tau}{1 + N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} d\tau}} - 1 \right] dt = \\
 &= \int_0^{t_k} \frac{c(t)N_0 e^{\int_0^t u d\tau}}{1 + N_0 \int_0^t \varepsilon(\tau) e^{\int_\tau^t u d\xi} d\tau} \cdot \frac{\int_0^t \Delta u d\tau - \frac{N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} \int_\tau^t \Delta u d\xi d\tau}{1 + N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} d\tau}}{1 + \frac{N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} \cdot \int_\tau^t \Delta u d\xi d\tau}{1 + N_0 \int_0^t \varepsilon e^{\int_\tau^t u d\xi} d\tau}} dt
 \end{aligned}$$

From here, the gradient functional for solving optimization problems and determining the optimal fish rearing policy is easily determined. Now let us consider the numerical calculations from the model population by law (7)-(16).

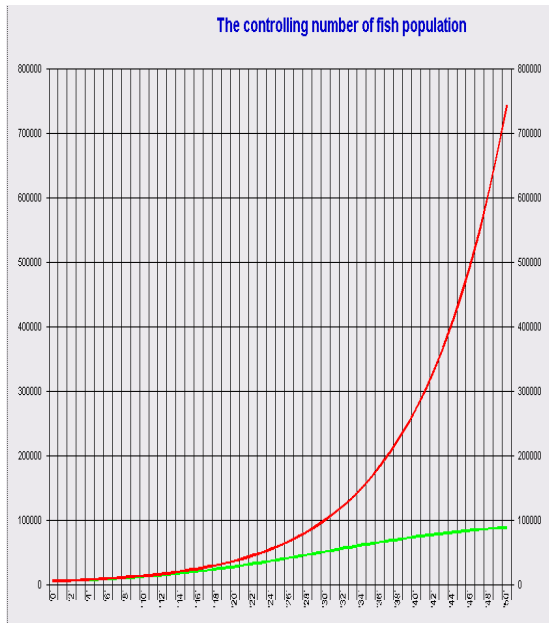


Fig.5. Numerical calculations from a model population

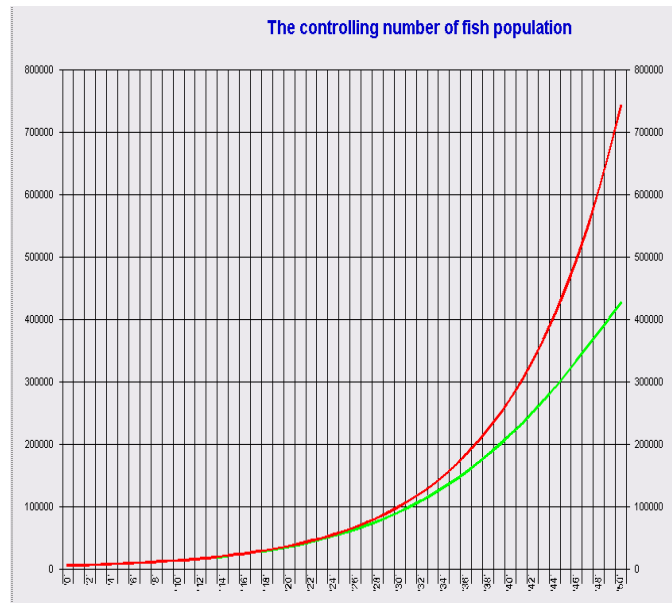


Fig.6. Numerical calculations from a model population

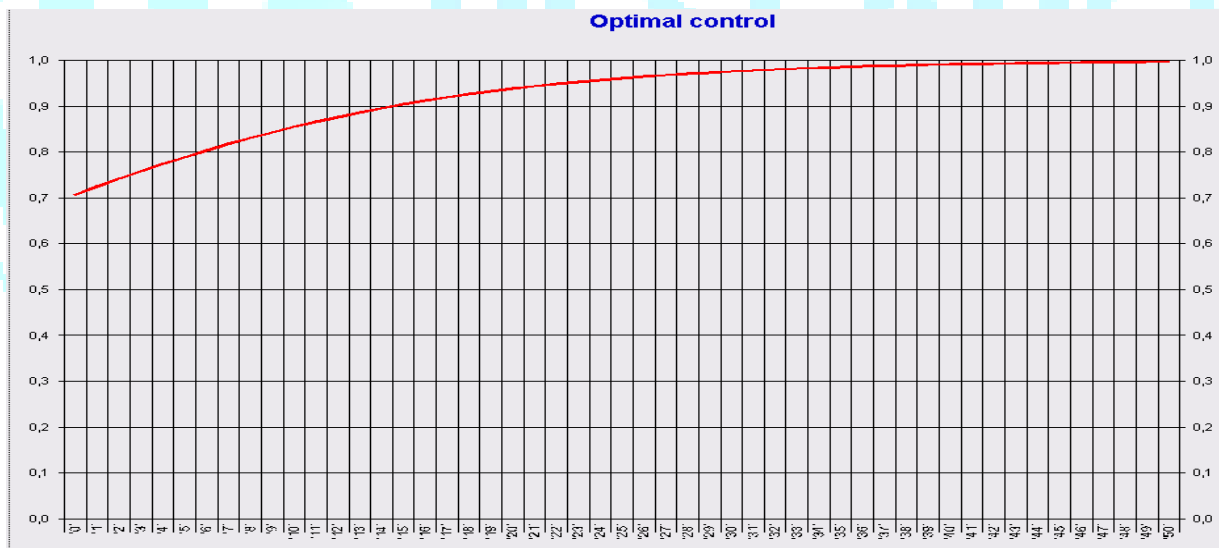


Fig.7. Numerical calculations from a model population

## References

1. «Pond fish farming" by N.I. Bogdanov, A.Y. Asanov. - 3rd ed. Penza, 2011. – p.89.
2. «Computer Modeling of Reservoir Ecosystems", F.S. Komilov, Dushanbe. 2010, p.237.
3. Nimatova M., Odinaeva S.A. Optimization of the process of growing fish in bioponds.//Bulletin of the Tajik National University. Series of Natural Sciences. 2013. №1-2(106). P. 43-47.ISSN 2074-1847
4. «Mathematical modeling of reservoir ecosystems», F.S. Komilov, V.P. Parkhomenko , Computational Center IM.A. Dorodnitsyn RAS. Moscow 2003, p.46