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Some Tasks Resulting in Differential Equations

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Abstract: In this paper, problems are presented, the solution of which leads to differential equations. These tasks show that differential equations are an effective mathematical apparatus through which the main tasks of natural science are realized.

Keywords: Differential equations, tangent slope, partial solution, arbitrary constant, derivative, initial velocity, initial conditions

In various fields of science and technology, problems are often considered, the solution of which is reduced to one or more equations containing, in addition to variables, the desired functions, also the derivatives of these desired functions, such equations are called differential. Differential equations relate the desired function to its derivatives of various orders and independent variables. Differential equations are unique in content and universal in their use as a means of knowing the world.

Differential equations contribute to a more objective understanding of a phenomenon, their effective use and increase the reliability of the results obtained.

If in the differential calculus for a given function there is its derivative, and in the integral calculus for the derivative a function is found that is primitive for this derivative, then when studying the theory and practical solution of differential equations, neither the function nor its derivative is given, but an equation (or several equations) that connects them is given. Mathematical analysis fully implements itself in applied problems through differential equations.

This is natural, since the derivative of the function defines such concepts as the angular coefficient of the tangent to the curve, the instantaneous speed of movement, the current strength in the electric acceleration, the current movement in the circuit, and so on. [3, 78] Consider several problems leading to the differential equation

Task1. The body is thrown vertically upwards with an initial velocity of v_0. Determine the law of motion of a body if its position at the initial moment t = 0 is s_0, assuming that the body moves only under the influence of gravity. [1,53]

Solution: As is known from the physics course, under the influence of gravity, a body moves with a constant acceleration equal to g. We know that the acceleration of the motion of a material point is expressed by the second-order derivative of the path s in time t, so the differential equation in this case has the form:

$$m\frac{d^2s}{dt^2} = -gm$$

or, after a reduction by mass m,
$$\frac{d^2s}{dt^2} = -g. \qquad (1)$$

According to the definition of the second derivative $s'' = (s')' = v' = -g$, therefore, reasoning in the same way as when solving the first problem, we will have that the function $v = s'$ there is a primitive with respect to the function $(-g)$, that is

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$v = s' = \int(-g)dt = -gt + C_1,$ $v = s' = -gt + C_1$ (2)

From the last equality it follows that s is the primitive of the function $(-gt + C_1)$

So $s = \int(-gt + C_1)dt = -g\int tdt + C^1 \int dt = -\frac{gt^2}{2} + C_1 t + C_2$

Where C_1 and C_2 are arbitrary constants. So, the solution of the differential equation is the function

$s = -\frac{gt_2}{2} + C_1 t + C_2$ (3)

Where C_1 and C_2 are arbitrary constants. Thus, the solution contains two arbitrary constants C_1 and C_2

According to the condition of the problem, at the initial moment of time t = 0, the body was at the height of s_0, so from the formula we have

$s(0) = S_0 = -\frac{g \cdot 0}{2} + C_1 \cdot 0 + C_2$ (4)

so $C_2 = s_0$ On the other hand, at the initial moment of time t=0, the body had an initial velocity v_0, so the formula follows :

$v_0 = -g \cdot 0 + C_1$ (5)

$C_1 = v_0$ So, substituting the values C_1 and C_2 from the equalities and the formula, we get a solution to the problem:

$$s = -\frac{gt^2}{2} + v_0 t + s_0$$

Task2. Let's assume that at each moment of time t, the velocity f(t) of a point moving along the $0x$ axis is known, where f(t) is a function continuous on (a, b). In addition, we will assume that the abscissa of the point x_0 is known, at some certain point in time $t = t_0$.It is required to find the law of motion of the point, that is, the dependence of the abscissa of the moving point on time.[2,34]

Solution: The position of a point is determined by one coordinate x and the task is to express x as a function of t. Taking into account the mechanical meaning of the first derivative, we obtain the equality $\frac{dx}{dt} = f(t)$ (1)

As is known from integral calculus

$x(t) = \int_{t_0}^{t} f(t) + C \;\; a < t < b$ (2)

Where the upper limit of the integral is variable, the lower t_0 is some fixed number of (a, b).C is an arbitrary constant. Since an arbitrary constant is included in formula (2), a definite law of motion of the point has not yet been obtained. Let us select from the set of values (2) the motion in which the moving point occupies a given position x_0 at a given time t_0

$X_0 = \int_{t_0}^{t} f(t)dt + C$ $C = x_0$

Which together with (2) gives the desired law of motion of the point:

$$x(t) = \int_{t_0}^{t} f(t)dt + x_0 (a < t < b).$$

From the examples considered, the specifics of solving differential equations are visible. Their solution is an infinite set of functions, from which solutions are obtained by setting initial conditions.

For example, in task 1, the following initial conditions were set: for the moment t=0, the initial position was set $s(0) = s_0$ (that is, the height of the body at the initial moment of time) and the initial velocity $v(0) = s'(0) = v_0$. This allowed from the general solution

$$s = -\frac{gt^2}{2} + C_1 t + C_2$$

find the private solution we are interested in $s = -\frac{gt^2}{2} + v_0 t + s_0$,

which is a solution to the problem under consideration.

Differential equation of harmonic oscillations $y'' = -k^2 y$. This equation is a second-order equation.

All solutions of this equation are exhausted by functions of the form

$y(x) = Asin(kx + \theta)$, where A and θ are arbitrary constants.

In the case under consideration, the general solution includes two arbitrary constants, therefore, in order to find a particular solution (so, the solution of the problem), initial conditions should be set, namely, at the initial moment, not only the value of the function should be set, but also the value of its derivative. For example, when considering the problem of a spring pendulum at the initial moment of time t=0, the initial deviation of the load from the equilibrium position $y(0)=y_0$ is given, as well as the speed with which this load moves at the initial moment t=0, so the initial velocity $y'(0) = v_0$

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