

Mathematical Models of Differential Transformer Sensors Tuned to the Excitation Winding Resonance

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Abstract: The analysis of the developed mathematical models of the new differential transformer sensors whose excitation circuit is adjusted to the resonance of voltages or currents showed that the sensitivity of the new sensors adjusted to the resonance of the excitation circuit is higher than the sensitivity of the sensors adjusted to the current resonance, and the original value of the excitation circuit is higher than the sensitivity of the sensors adjusted to the resonance of the currents. This showed that the sensitivity of the sensors is as many times higher than the sensitivity of the sensors tuned to voltage resonance. It was found that the change in the value and character of the load resistance in two identical new differential transformer sensors whose excitation circuit is tuned to voltage and current resonance, respectively, leads to a greater change in the current value in the excitation winding of the DTS tuned to voltage resonance.

Keywords: differential transformer sensor, mathematical model, inductance, capacitor, resonance of voltages, resonance of currents, sensitivity, active power, full power, circuit originality, working air gap, magnetic resistance; magnetic capacity.

Various models, in particular, differential transformer sensors (DTS), are widely used to obtain information about device displacements in controlled objects in technological processes and automatic production control systems. [1,2]. These DTSs have a nonlinear conversion function and a relatively low sensitivity in the measuring range, according to a comparison of their key properties. [3,4]. The quality indicators of the control process are reduced when DTSs with these flaws are used in automatic control systems.

In order to eliminate the above-mentioned shortcomings, a new construction of displacement measuring DTS was developed at the Tashkent State Transport University (1- pacm) [5]. In this DTS, the values of the two working air gaps between the long ferromagnetic rods located at the edge and in the middle are made to vary along the length of the chain according to the following laws:

$$\delta_{x1} = \delta_{min} + \frac{1}{\mu h} (X_M + x)^2, \quad (1)$$

$$\delta_{x2} = \delta_{min} + \frac{1}{\mu h} (X_M - x)^2, \quad (2)$$

where μ – is the relative magnetic absorbance of the magnetic material of ferromagnetic rods, and the corresponding geometric dimensions of the magnetic circuit are shown in Fig. 1, a.

Although the operating principle of the new DTS does not differ from the operating principle of its prototype [3], in the new DTS, when the sizes of the working air gaps are chosen to vary along the length of the magnetic circuit according to expressions (1) and (2), respectively, both inductances in each of the excitation and measurement windings changes linearly when the coordinate of the EE changes, while the total inductances in each of the excitation and measurement windings are kept constant throughout the displacement range of the EE. As a result, the resonance mode in the excitation circuit is maintained at any coordinate of the EE (Fig. 1, b), and the static description of the DTS is linear.

In order to dramatically increase the sensitivity of displacement measuring DTSs, their excitation and measurement circuits or one of them is tuned to resonance. [6,7]. In this case, if the measurement circuit of the DTSs can only experience voltage resonance, voltage or current resonance modes may emerge in the excitation circuit.

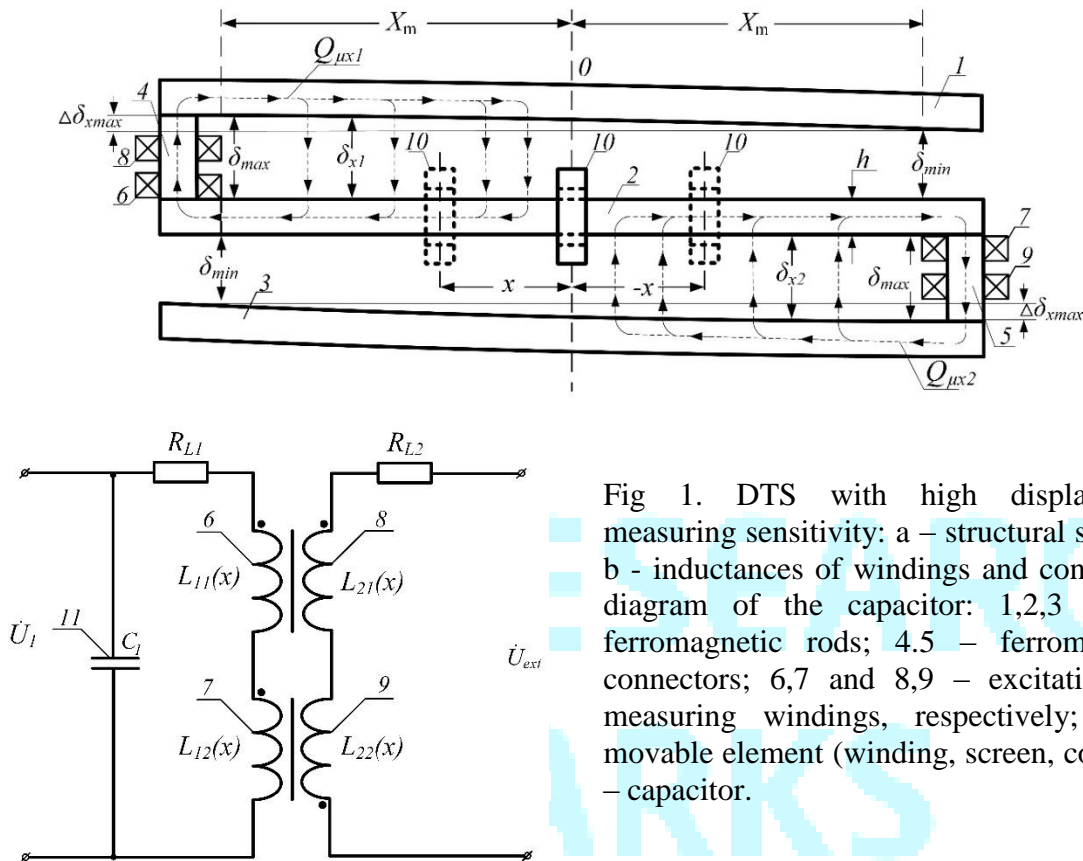


Fig 1. DTS with high displacement measuring sensitivity: a – structural scheme; b - inductances of windings and connection diagram of the capacitor: 1,2,3 - long ferromagnetic rods; 4,5 – ferromagnetic connectors; 6,7 and 8,9 – excitation and measuring windings, respectively; 10 – movable element (winding, screen, core); 11 – capacitor.

In the case of DTSs, where the driven element is investigated in a long ferromagnetic rod and is composed of a measuring winding tightly wound around the rod, the originality of the measuring winding (the ratio of the coil's inductive resistance to its active resistance) is low, making it ineffective to induce a resonance mode in it. Therefore, in the dissertation, we will focus on the resonance modes of DTS that are formed only in the excitation winding.

Initially, when the measuring winding is connected to a load with high resistance, i.e., the measuring winding is operating in a mode close to the pure operating mode, we do not take into account the influence of the current in the measuring winding on the electromagnetic processes in the excitation winding in the study of resonance modes in the excitation circuit of the DTS [8]. To adjust the DTS excitation circuit to the resonance of voltages (currents), the value of the capacitor connected in series (parallel) to the excitation circuit is calculated using the following formulas, based on the values of the inductance of the excitation circuit and the frequency of the source voltage, as well as the resonance condition. [8]:

$$C_{s-s} = 1/(\omega_r^2 L_s), \quad (3)$$

$$C_{p-l} = L_s/(\omega_r^2 L_s^2 + R_s^2), \quad (4)$$

where R_s , L_s , C_{s-s} , C_{p-l} – active resistance, inductance of the excitation winding, capacitance of the capacitor connected in series or parallel to it, respectively; ω_r – is resonance frequency.

As it is known [9], in the resonance of voltages (v.r.) and currents (c.r.), the current and power in the circuit are equal to:

$$I_{v.r.} = U/R_s, \quad (5) \quad I_{c.r.} = UR_s/(R_s^2 + X_{s,L}^2); \quad I_{c.r.(L)} = U/\sqrt{R_s^2 + X_s^2}, \quad (6)$$

$$S_{v.r.} = P_{v.r.} = U^2/R_s, \quad (7) \quad S_{c.r.} = P_{c.r.} = U^2R_s/(R_s^2 + X_{s,L}^2). \quad (8)$$

The sensitivity of the new DTS, made in the form of a measuring rod of the investigated moving element, is found as follows:

$$K = \frac{dE_2}{dx} = \omega w_s w_m I_1 \frac{C_{\mu n0}}{\Delta} 2x, \quad (9)$$

where $\Delta = 1 + Z_{\mu 0} C_{\mu c0} X_M + Z_{\mu c} C_{\mu c0} X_m^2, [-]$; $C_{\mu c0}$ – is the pogan (comparative) value of the magnetic capacity between two long ferromagnetic rods in $\delta min, [H/m^{-1}]$; $Z_{\mu c}$ – pogan value of magnetic resistance of long ferromagnetic rods, $H[H^{-1} \cdot m^{-1}]$; $Z_{\mu 0}$ – magnetic resistance of ferromagnetic connectors, $[H^{-1}]$; w_s, w_m – the number of windings in the excitation and measurement windings, respectively.

The sensitivities of the new DTSs adjusted to the resonance of the excitation voltages (v.r.) and currents (c.r.) and their relative values per unit of power consumed from the source are found to be, respectively,

$$K_{v.r.} = \omega w_s w_m \frac{2U_s C_{\mu c0}}{R_s \Delta}, \quad (10) \quad K_{c.r.} = \omega w_s w_m \frac{2U_s C_{\mu c0}}{\Delta \sqrt{R_s^2 + X_{s,L}^2}}. \quad (11)$$

$$K_{v.r.}^* = \omega w_s w_m \frac{2C_{\mu c0}}{U_s \Delta}, \quad (12) \quad K_{c.r.}^* = \omega w_s w_m \frac{2C_{\mu c0}}{U_s \Delta} \sqrt{1 + (X_{s,L}/R_s)^2}, \quad (13)$$

where U_s – is the source voltage; $X_{s,L}$ – is the inductive resistance of the excitation winding.

A mutual comparison of the expressions (10) and (11) and (12) and (13) shows that the sensitivity of the DTS adjusted to the resonance of the excitation circuit voltages at the same values of the parameters R_s, L and C of the DTS excitation circuit and the source voltage is greater than the sensitivity of the DTS adjusted to the current resonance $\sqrt{1 + (X_{s,L}/R_s)^2}$ times higher, and the sensitivity of the DTS tuned to the current resonance is greater than the sensitivity of the DTS tuned to the voltage resonance when taken in relation to the unit of power consumed by the source $\sqrt{1 + (X_{s,L}/R_s)^2}$ times higher, for example, in the DTS with the originality of the excitation chain equal to 10, the quantities compared above differ by $\sqrt{1 + (X_{s,L}/R_s)^2} = 10,05$

Now we analyze the electromagnetic processes occurring in the new DTS, where the measuring winding is connected to a resistive load Z_{up} and the excitation winding is tuned to resonance, and the excitation and measuring windings are inductively connected to each other (Fig. 2)

For the variant of the DTS presented in Fig. 2, the excitation circuit is adjusted to voltage resonance, based on Kirchhoff's second law, we construct the following balance equations (in this case, the branch indicated by the continuous line in the scheme is not taken into account) [9]:

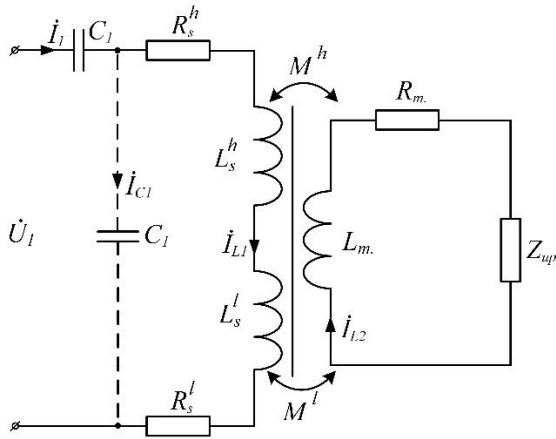


Figure 2. Exchange scheme of a new DTS electromagnetic circuit with the measuring winding connected to a resistive load $Z_{\text{ЮК}}$ and the excitation winding tuned to resonance

$$\dot{U}_1 = [R_s + j(\omega L_s - 1/\omega C_1)]\dot{I}_{L1} - j\omega M(x)\dot{I}_{L2}, \quad (14)$$

$$(R_m + Z_{up} + j\omega L_m)\dot{I}_{L2} - j\omega M(x)\dot{I}_{L1} = 0, \quad (15)$$

where $R_s = R_s^h + R_s^l$; $L_s = L_s^h + L_s^l$; $(x) = M^h - M^l = w_s w_m \frac{C_{\mu c0}}{\Delta} (X_m + x) - w_s w_m \frac{C_{\mu c0}}{\Delta} (X_m - x) = 2w_s w_m \frac{C_{\mu c0} x}{\Delta}$ - is the mutual inductance between the excitation and the measuring windings, whose windings are connected in series and inductively opposite to each other.

Taking into account that $\omega L_s - \frac{1}{\omega C_1} = 0$ in voltage resonance, we solve equations (14) and (15) together with respect to currents \dot{I}_{L1} and \dot{I}_{L2} and obtain their following values:

$$\dot{I}_{L1} = \frac{\dot{U}_1}{R_s + \frac{[\omega M(x)]^2 (R_m + Z_{up})}{(R_m + Z_{up})^2 + (\omega L_m)^2} - j \frac{[\omega M(x)]^2 \omega L_m}{(R_m + Z_{up})^2 + (\omega L_m)^2}} = \frac{\dot{U}_1}{Z_{as}}, \quad (16)$$

$$\dot{I}_{L2} = \frac{j\omega M(x)\dot{U}_1}{R_s(R_m + j\omega L_m + Z_{up}) + [\omega M(x)]^2}, \quad (17)$$

where $Z_{as} = R_s + \frac{[\omega M(x)]^2 (R_m + Z_{up})}{(R_m + Z_{up})^2 + (\omega L_m)^2} - j \frac{[\omega M(x)]^2 \omega L_m}{(R_m + Z_{up})^2 + (\omega L_m)^2}$ is the input resistance of the DTS electromagnetic circuit to the source.

The analysis of the expressions (16) and (17) generated for the currents in DTS circuits in which the excitation circuit is tuned to voltage resonance and the measuring circuit is connected to the load shows that $I_{L2} = 0$ when $Z_{up} \rightarrow \infty$, and the current in the excitation circuit is equal to $\dot{I}_{L1(p)} = \frac{\dot{U}_1}{R_s}$.

At $Z_{up} \neq 0$, Z_{as} resistance is active $\left(\frac{[\omega M(x)]^2 (R_m + Z_{up})}{(R_m + Z_{up})^2 + (\omega L_m)^2}\right)$ and the reactance $\left(\frac{[\omega M(x)]^2 \omega L_m}{(R_m + Z_{up})^2 + (\omega L_m)^2}\right)$ founders appear.

These components, on the one hand, cause a change in the amplitude and phase of the excitation current, and on the other hand, the presence of a reactive component causes a change in the specific frequency of the excitation circuit tuned to resonance, as a result of which the circuit deviates to some extent from the resonance mode.

Now, for the version of the DTS excitation circuit tuned to current resonance, we draw up the following balance equations based on Kirchhoff's laws (where the capacitor in the common branch of the circuit shown in Figure 2 is not taken into account, but the branch shown by the continuous line is taken into account):

$$\dot{I}_1 = \dot{I}_{L1} + \dot{I}_{C1}, \quad (18) \quad \dot{U}_1 = -jX_C \dot{I}_{C1}, \quad (19)$$

$$\dot{U}_1 = (R_s + j\omega L_s) \dot{I}_{L1} - j\omega M(x) \dot{I}_{L2}, \quad (20)$$

$$(R_m + Z_{up} + j\omega m.) \dot{I}_{L2} - j\omega M(x) \dot{I}_{L1} = 0, \quad (21)$$

Finding the current \dot{I}_{C1} from equation (19), and the current \dot{I}_{L1} from equation (20), putting their values in (18), we form the following expression for the current \dot{I}_1 in the unbranched part of the circuit:

$$\dot{I}_1 = \frac{R_s \dot{U}_1}{R_s^2 + (\omega L_s)^2} + j \frac{\omega C_1 [R_s^2 + (\omega L_s)^2 - \frac{L_s}{C_1}] \dot{U}_1}{[R_s^2 + (\omega L_s)^2] X_C} + j \frac{\omega M(x) \dot{I}_{L2}}{R_s + j\omega L_s}, \quad (22)$$

If it is taken into account that the DTS excitation circuit under investigation is tuned to current resonance, then the second addition on the right side of equation (22) is equal to zero. This can be confirmed by putting the value of $\omega_r = \frac{1}{\sqrt{L_s C_1}} \sqrt{\frac{L_s / C_1 - R_s^2}{L_s / C_1}}$ instead of the angular frequency ω in the second adder.

Taking into account the above and solving equations (20) and (21) together, we find the following values of currents \dot{I}_1 , \dot{I}_{L1} and \dot{I}_{L2} :

$$\dot{I}_1 = \frac{R_{1s} \dot{U}_1}{R_{1s}^2 + (\omega L_{1s})^2} - j \frac{[\omega M(x)]^2 (R_{1s} - j\omega L_{1s}) \dot{U}_1}{[R_{1s}^2 + (\omega L_{1s})^2] [(R_{1s} + j\omega L_{1s})(R_{L2} + Z_{up} + j\omega L_2) + [\omega M(x)]^2]}, \quad (23)$$

$$\dot{I}_{L1} = \frac{(R_{L2} + Z_{up} + j\omega L_2) \dot{U}_1}{(R_{1s} + j\omega L_{1s})(R_{L2} + Z_{up} + j\omega L_2) + [\omega M(x)]^2}, \quad (24)$$

$$\dot{I}_{L2} = \frac{j\omega M(x) \dot{U}_1}{(R_{1s} + j\omega L_{1s})(R_{L2} + Z_{up} + j\omega L_2) + [\omega M(x)]^2}, \quad (25)$$

It should be said that when $Z_{up} \rightarrow \infty$, equations (23)-(25) change to the form $\dot{I}_{1(r)} = \frac{R_{1s} \dot{U}_1}{R_{1s}^2 + (\omega L_{1s})^2}$, $\dot{I}_{L1(r)} = \frac{\dot{U}_1}{R_{1s} + j\omega L_{1s}}$ and $\dot{I}_{L2} = 0$, respectively.

The following functions can be used for the DTS tuned to voltage (current) resonance (v.r. and c.r.) when evaluating the deviation of the current in the excitation winding from its value in the resonance mode depending on the $Z_{юк}$ and the coordinate of the measuring coil with the DTS driven element:

$$I_{L1(v.r.)}^* = \frac{I_{L1}}{I_{L1(r)}} = f_{v.r.}(x, Z_{up}) = \frac{R_{1s}}{\sqrt{\left[R_{1s} + \frac{[\omega M(x)]^2 (R_{L2} + Z_{up})}{(R_{L2} + Z_{up})^2 + (\omega L_2)^2} \right]^2 + \left[\frac{[\omega M(x)]^2 \omega L_2}{(R_{L2} + Z_{up})^2 + (\omega L_2)^2} \right]^2}}. \quad (26)$$

$$I_{L1(c.r.)}^* = f_{c.r.}(x, Z_{up}) = \sqrt{\frac{[(R_{L2} + Z_{up})^2 + (\omega L_2)^2] [R_{1s}^2 + (\omega L_{1s})^2]}{[R_{1s} (R_{L2} + Z_{up}) + [\omega M(x)]^2 - \omega^2 L_{1s} L_2]^2 + [R_{1s} \omega L_2 (R_{L2} + Z_{up})]^2}}}. \quad (27)$$

$\mu_{av} = 1810$ (the average value of the magnetic induction in the range of $0,4 \div 1,5 T$ in the magnetic conductor assembled from 1512 grade electrotechnical steel tins; $b = 0,02 m$; $h = 0,01 m$; $\delta = 0,02 m$; $X_m = 0,075 m$; $w_s = 500$; $w_m = 50$. The diameter of the electric conductor used for the excitation winding

is $d = 0,2 \text{ mm}$ and the Pogon value of its resistance is $R = 0,152 \text{ } \Omega/m$, and for the measuring coil, $d = 0,2 \text{ mm}$ and $R = 0,572 \text{ } \Omega/m$, respectively. Measured values of circuit parameters: $R_s = 13,7 \text{ } \Omega$

$$R_m = 2,6 \text{ } \Omega; L_s = 0,19 \text{ H}; L_m = 9,5 \cdot 10^{-4} \text{ H}; M_l = \frac{M}{X_m} = 1,27 \cdot 10^{-2} \text{ H/m}.$$

Equations (26) and (27) and $I_{L1(v.r.)}^* = f_{v.r.}(x, Z_{up})$ and $I_{L1(c.r.)}^* = f_{c.r.}(x, Z_{up})$ function graphs (Fig. 3) analysis shows that the change in the value and character of the load resistance in two identical new DTSs, whose excitation winding are tuned to voltage and current resonance, respectively, leads to a greater change in the current value in the excitation winding of the DTS tuned to voltage resonance.

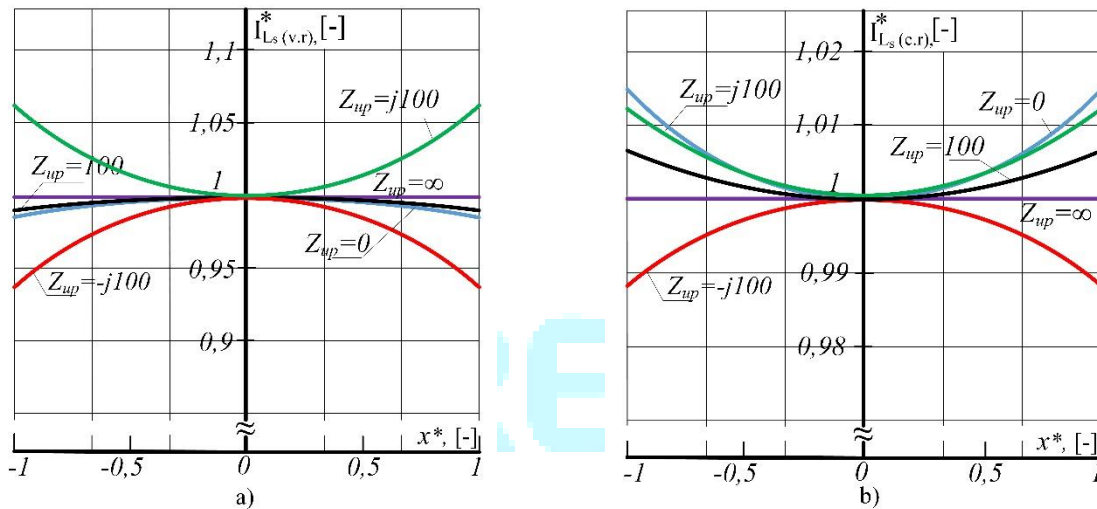


Figure 3. DTSs adjusted to resonances of voltages (a) and currents (b) of the investigated displacement measuring and excitation winding $I_{L_s(v.r.)}^* = f_{v.r.}(x, Z_{up})$ and $I_{L_s(c.r.)}^* = f_{c.r.}(x, Z_{up})$ graphs of functions

Thus, in this article, the mathematical models of the new DTS, whose excitation winding is adjusted to the resonance of voltages or currents, are formed in the form of analytical equations (10)-(13), (16), (17), (23)-(27).

With their help, it will be possible to compare the technical capabilities of the new DTS, whose excitation winding is tuned to the resonance of voltages or currents, and to theoretically study their specifications.

The analysis of mathematical models developed for DTSs whose excitation circuit is tuned to voltage or current resonance showed that the sensitivity of DTS tuned to voltage resonance is $\sqrt{1 + X_{s,L}^2/R_s^2}$ times higher, and the sensitivity of the DTD adjusted to the current resonance is $\sqrt{1 + X_{s,L}^2/R_s^2}$ times higher than the sensitivity of the DTS adjusted to the voltage resonance, for example, the excitation circuit in a DTS with originality equal to 10, the quantities compared above differ by $\sqrt{1 + X_{s,L}^2/R_s^2} = 10,05$.

A change in the value and character of the load resistance in two identical new DTSs, whose excitation winding are tuned for voltage and current resonance, respectively, leads to a greater change in the value of the current in the excitation winding of the voltage-resonant DTS.

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