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Article

Interval Potential Method for Solving Transportation Problems Using Mathematical Programming

Dilafruz Khamroeva[1]

1. Navoi State University, Navoi, Uzbekistan
* Correspondence: hdilafruz285@mail.ru

Abstract: This study explores the interval variant of the potential method as an innovative approach for solving transportation problems within mathematical programming. Traditional methods often fail to address the complexities arising from parameter uncertainties, creating a knowledge gap in deriving reliable solutions under varying conditions. To bridge this gap, the interval potential method is proposed, utilizing interval matrices to define constraints and feasible solutions. The methodology involves constructing the initial transportation plan using the northwest corner method and applying interval analysis to account for data variability. A structured algorithm calculates directional potentials and checks the plan's acceptability, iteratively adjusting for optimal results. Numerical simulations demonstrate the robustness of the proposed method in solving transportation problems with uncertain parameters. Results confirm that this approach identifies optimal interval solutions while maintaining computational efficiency. The implications extend to various fields requiring reliable transportation and logistics optimization under uncertain conditions, such as supply chain management and resource allocation. This work contributes to the broader application of interval analysis in mathematical programming, providing a scalable solution for real-world challenges.

Keywords: Interval analysis, Potential method, Transportation problem, Mathematical programming, Interval matrices

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1. Introduction

One of the main conditions for further economic development is the implementation of quantitative analysis based on mathematical methods and new computer technologies, and the adoption of economic decisions based on this. Mathematical programming is the science that teaches the methods that are useful in carrying out such tasks.

The components of mathematical programming are linear, nonlinear, and dynamic programming. The problem of linear programming in the form of creating an optimal transportation plan, allowing for the minimization of total mileage, was first posed by the Soviet economist A.N. Tolstoy in 1930. Since 1939, L.V. Kantorovich has been regularly researching problems of linear programming and developing general methods for their solution.

In 1941, Hichkok brought up the issue of transportation. In 1949, D. Dansig published the simplex method, which is the main method for solving linear programming problems. In the same year, L.V. Kantorovich, together with M.K. Gavurin, developed the potential method used to solve the transport problem [6].

In 1951, Kuhn and Tucker published necessary and sufficient optimality conditions for solving nonlinear problems. In addition, B. Egervari, V.S. Nemchinov, A.L. Lure, D.B.

Yudin, R. Bellman, L. Ford, S. Gass, and others made a significant contribution to the development of mathematical programming [7].

It is known that the transport problem [7] serves as a model for many practical problems in the sense of a simple reduction of the latter to one variant or another. In turn, the transport problem associated with bilateral constraints is of particular interest [2]. Considering the corresponding operations and algorithms within the framework of interval analysis instead of bilateral constraints, we can discuss the problem of finding an interval matrix containing a set of optimal designs. If part or all of the problem is specified within a range of parameters, then the required interval matrix should contain an optimal plan for any degenerate problem, i.e., any real problem arising from assigning values within the given intervals.

2. Materials and Methods

Let the cost of transporting the same cargo unit from the supplier $\mathbf{d}_{ij}$ (item A_i) to the consumer j (item C_j) be $\mathbf{x}_{ij}=[\underline{x}_{ij},\ \overline{x}_{ij}]$; i is the amount of cargo delivered from the supplier to the second consumer; a_i is the amount of cargo at the point; A_i is the requirement for the product in the paragraph.

Therefore, the following interval problem arises

$$\left|\mathbf{F}\right| = \left|\sum_{i=1}^{m}\sum_{j=1}^{n}\mathbf{d}_{ij}\,x_{ij}\right| \to \min, \quad (1)$$

$$\sum_{j=1}^{n}\overline{x}_{ij} = a_i \quad (i=\overline{1,\ m}), \quad (2)$$

$$\sum_{i=1}^{m}\mathbf{x}_{ij} = \mathbf{b}_j \quad (j=\overline{1,\ n}), \quad (3)$$

$$\mathbf{x}_{ij} \subseteq \mathbf{w}_{ij} \quad (i=\overline{1,\ m},\quad j=\overline{1,\ n}), \quad (4)$$

$$\sum_{i=1}^{m}a_i = \sum_{j=1}^{n}\overline{b}_j, \quad (5)$$

where $\mathbf{X}=(x_{ij})=([\underline{x}_{ij},\ \overline{x}_{ij}])$ is the interval matrix of transport, $\mathbf{W}=(w_{ij})=([\underline{w}_{ij},\ \overline{w}_{ij}])$ is the interval matrix of transport constraints

$$\sum_{i=1}^{m}\underline{w}_{ij} \le \underline{b}_j \le \overline{b}_j \le \sum_{i=1}^{m}\overline{w}_{ij}, \qquad (j=\overline{1,n}), \quad (6)$$

Definition. The support plan for problems 1 (1) - (6) is a $\mathbf{X}$ matrix that satisfies conditions (2) - (6).

It should be noted that the problem (1) - (5), under the condition (6), can be interpreted as a model for a situation where we know the claims and believe that we should satisfy them to the greatest extent possible.

In general, instead of (6), from a mathematical point of view, any choice from the relations

$$\underline{b}_j \ne \underline{w}_{ij},\ \ \overline{w}_{ij} \ne \overline{b}_j \quad (j=\overline{1,\ n}). \quad (7)$$

However, the cases (7) are analyzed below, based on traditional criteria for solving the transport problem, including consideration of the uncertainty interval of the parameters:

Theorem 1. Any degenerate problem corresponding to the interval problem (1) - (6) can be solved.

Evidence

$$\sum_{j=1}^{n}\underline{b}_j = \underline{s}, \ \sum_{i=1}^{m}a_i = \sum_{j=1}^{n}\overline{b}_j = \overline{s}.$$

Then, according to the condition of theorem $\underline{x}_{ij} = \overline{x}_{ij} = x_{ij}$, and for the quantities $x_{ij} \in [(a_i\underline{b}_j)/\underline{s}, (a_i\overline{b}_j)/\overline{s}]$, we have:

$$\sum_{j=1}^{n}x_{ij} \geq \sum_{j=1}^{n}(a_i\underline{b}_j)/\underline{s} = \frac{a_i}{\underline{s}}\sum_{j=1}^{n}\underline{b}_j = a_i, \qquad (i=\overline{1, m}),$$

$$\sum_{j=1}^{n}x_{ij} \leq \sum_{j=1}^{n}(a_i\overline{b}_j)/\overline{s} = \frac{a_i}{\overline{s}}\sum_{j=1}^{n}\overline{b}_j = a_i, \qquad (i=\overline{1, m}),$$

$$\sum_{i=1}^{m}x_{ij} \geq \sum_{i=1}^{m}(a_i\underline{b}_j)/\underline{s} = \frac{\underline{b}_j}{\underline{s}}\sum_{i=1}^{m}a_i = \underline{b}_j\frac{\overline{s}}{\underline{s}} \geq \underline{b}_j, \quad (j=\overline{1,n}),$$

$$\sum_{i=1}^{m}x_{ij} \leq \sum_{i=1}^{m}(a_i\overline{b}_j)/\overline{s} = \frac{\overline{b}_j}{\overline{s}}\sum_{i=1}^{m}a_i = \overline{b}_j\frac{\underline{s}}{\overline{s}} \leq \overline{b}_j, \quad (j=\overline{1,n}).$$

Thus, for each set of specified parameters from the given intervals, there exists a nonempty set of Q solutions corresponding to constraints (2) - (4). Moreover, according to the properties of (4) and (6), the polytop Q is a finite set of $m \times n$ Euclidean measures, which indicates the existence of a solution to the problem (1) - (6).

3. Results

To determine the directional plan, we will use the northwest angle method algorithm with corresponding corrections to the parameter range. The formation of a table that allows for the preparation of a preliminary plan is carried out in accordance with the following rules:

1. Applications must be approved;
2. Only the amount of cargo that can remain in reserve can be sent to consumers.

If the interval representing the query in the composition of the tables has a left boundary, then due to the lack of incoming traffic, we will consider the left boundary equal to zero.

Endi 1°, 2° qoidalariga asoslanib, boshlang‘ich rejani aniqlash algoritmini quyidagi misol bilan tasvirlaymiz. Now, based on the rules of 1°, 2°, we will describe the algorithm for determining the initial plan with the following example.

For example. Let it be $m=2$, $n=3$, $a_1=10$, $a_2=8$, $\mathbf{b}_1=[2,5]$, $\mathbf{b}_2=[3,8]$, $\mathbf{b}_3=[4,5]$, $\mathbf{w}_{ij} \supseteq \mathbf{b}_{ij}$, $\forall i, j$.

1. Comparing $\mathbf{b}_1$ and a_1 gives $\mathbf{x}_{11}=[5,5]$, $\mathbf{x}_{13}=[0,0]$. Since it is $[10,\ 10]-[2,\ 5]=[5,\ 8]$, we assume that a_1 has a load left.

2. Comparison with the rest of the load at $\mathbf{b}_2$ a_1 $\mathbf{x}_{13}=[0,0]$, $[3, 8]-[5, 5]=[-2, 3]$ since the remaining request at $\mathbf{b}_2$ [0,3].

Thus, the basic plan of the problem looks as follows: $\mathbf{x}=\begin{pmatrix} [2, 5] & [5, 5] & 0 \\ 0 & [0, 3] & [4, 5] \end{pmatrix}$.

In the real case, the corresponding writing plan should be determined by many methods of a repetitive type, such as the simplex method, the potential method, etc. When considering intervals, the potential method turned out to be more convenient as the main real, as in this case the potentials can be differentiated themselves: the potentials from the suppliers' departure points can be real and the potentials (destinations) can be intermediate [8].

To describe the algorithm for solving the problem (1) - (6), we introduce some concepts.

Definition 2.3. Based on the location of the transportation problem in the interval settings, we refer to the following table 1.

Table 1.

	C_1	C_2	. . .	C_n		
A_1	$\frac{\mathbf{x}_{11}}{\mathbf{d}_{11}\mathbf{w}_{11}}$	$\frac{\mathbf{x}_{12}}{\mathbf{d}_{12}\mathbf{w}_{12}}$	. . .	$\frac{\mathbf{x}_{1n}}{\mathbf{d}_{1n}\mathbf{w}_{1n}}$		a_1
A_2	$\frac{\mathbf{x}_{21}}{\mathbf{d}_{21}\mathbf{w}_{21}}$	$\frac{\mathbf{x}_{22}}{\mathbf{d}_{22}\mathbf{w}_{22}}$	. . .	$\frac{\mathbf{x}_{2n}}{\mathbf{d}_{2n}\mathbf{w}_{2n}}$		a_2
⋮	. . .	. . .	. . .	. . .		⋮
A_m	$\frac{\mathbf{x}_{m1}}{\mathbf{d}_{m1}\mathbf{w}_{m1}}$	$\frac{\mathbf{x}_{m2}}{\mathbf{d}_{m2}\mathbf{w}_{m2}}$	. . .	$\frac{\mathbf{x}_{mn}}{\mathbf{d}_{mn}\mathbf{w}_{mn}}$		a_m
	$\mathbf{b}_1$	$\mathbf{b}_2$	. . .	$\mathbf{b}_n$		

4. Discussion

If, as a result of compiling the corresponding records plan using the northwest corner method, after comparing the request on C_j with the rest of the cargo on A_i, it is equal to one zero, then this element is called the base zero.

A cycle is a sequence of layouts with the following properties:

a) The first and last cells are the same;
b) Both adjacent cells are located in the same row;
c) Both adjacent cells are located in the same row;
g) There are no three cells in one row

$$\mathbf{w}_{ij}=[0, \max_{i,j}\{a_i, \bar{b}_j\}] \quad (i=\overline{1, m}\,; \ j=\overline{1, n}\ ) . \tag{7}$$

Algorithm of interval potential method:

1. Based on the data from problem (1) - (6), we construct model $\mathbf{x}_{ij}=[0, 0]$ ($i=\overline{1, m}$, $j=\overline{1, n}$).
2. We form the reference plan $\mathbf{X}=(\mathbf{x}_{ij})$ using the northwest corner method based on the rules 1^0, 2^0, where the same cells of the layout are directed to $|\mathbf{x}_{ij}|\neq 0$ or the set of bases where the main zeros are located.

3. We calculate the potential u_i, $\mathbf{v}_j$ $(i=\overline{1,m};\ j=\overline{1,n})$, so that the condition $|\mathbf{v}_j| \le |\mathbf{d}_{ij}|$ $(i=\overline{1,m},\ j=\overline{1,n})$ is met in any base cell of the model. To do this, we place $u_1=0$ and determine the consecutive remaining potentials.
4. We'll check the current plan for acceptability. If $u_i+|\mathbf{v}_j| \le |\mathbf{d}_{ij}|$ $(i=\overline{1,m},\ j=\overline{1,n})$ for all cells of the order, then if the plan is acceptable, move on to step 9.
5. To determine the beginning of the future traffic distribution cycle, we will look for a cell in the layout (p,q) will look like this

$$u_p+|\mathbf{v}_q|-|\mathbf{d}_{pq}|=\max_{i,j}|u_i+\mathbf{v}_j-\mathbf{d}_{ij}| \tag{8}$$

6. We delete all rows from the layout except for row p and column q, except for the main row. If there's a lot more to draw, repeat this process in the rest of the order.
7. From the interval $(p,\ q)$, we construct the cycle L. All cells except L must be primary.
8. Let the value of $\mathbf{x}_{ij}$ in the cell with the base zero be zero $t=\min_{i,j}\left\{\min\{|\underline{x}_{ij}|,|\overline{x}_{ij}|\}\right\}$, where the minimum is taken for all cells located in all locations of the cycle. We organize the "load shift over the recalculation cycle":
   a. For all odd intervals, we multiply the transport by t as follows:

   $$\mathbf{x}_{ij}:=\mathbf{x}_{ij}+[t,\ t], \tag{9}$$

   b. The transport across all intervals decreases by t as follows:

   $$\mathbf{x}_{ij}:=\mathbf{x}_{ij}-[t,\ t]. \tag{10}$$

In this case, all zeros obtained as a result of the recalculation according to (9) and (10) are primary except for the first. Thus, in the newly obtained plan, there will be $n+m-1$ basic intervals.

Next, we'll take step 3.

9. Write the optimal plan obtained as a result from the model in the form i, j, $\mathbf{x}_{ij}$ for all intervals $|\mathbf{x}_{ij}|>0$. For some uncertainty in the parameters, we calculate the cost of this plan, taken as an interval describing the maximum and minimum cost of transportation.

5. Conclusion

The algorithm of the Potencial method is structured and consists of the following parts:

1. "Formation of the initial plan by the method" northwest corner";
2. Formation of the initial plan by the method of "northwest corner";
3. Counting potentials;
4. To check the current plan for its acceptability
5. Finding the cell - the beginning of the cycle of redistribution of traffic to the clock;;
6. Build a cycle;
   a. shift of load to recalculation cycle.

REFERENCES

[1] P. A. Mykland, "Financial options and statistical prediction intervals," *The Annals of Statistics*, vol. 31, no. 5, Oct. 2003, doi: 10.1214/aos/1065705113.

[2] P. Sevastjanov and L. Dymova, "A new method for solving interval and fuzzy equations: Linear case," *Inf Sci (N Y)*, vol. 179, no. 7, pp. 925–937, Mar. 2009, doi: 10.1016/j.ins.2008.11.031.

[3] R. R. Yager and V. Kreinovich, "Decision making under interval probabilities," *International Journal of Approximate Reasoning*, vol. 22, no. 3, pp. 195–215, Dec. 1999, doi: 10.1016/s0888-613x(99)00028-6.

[4] L. Dymova, P. Sevastjanov, and A. Tikhonenko, "A direct interval extension of TOPSIS method," *Expert Syst Appl*, vol. 40, no. 12, pp. 4841–4847, Sep. 2013, doi: 10.1016/j.eswa.2013.02.022.

[5] R. Rihm, "On a class of enclosure methods for initial value problems," *Computing*, vol. 53, no. 3–4, pp. 369–377, Sep. 1994, doi: 10.1007/bf02307387.

[6] G. Shepelyov and M. Sternin, "Methods for comparison of alternatives described by interval estimations," *International Journal of Business Continuity and Risk Management*, vol. 2, no. 1, p. 56, 2011, doi: 10.1504/ijbcrm.2011.040015.

[7] J. Zhu, Z. Ma, H. Wang, and Y. Chen, "Risk decision-making method using interval numbers and its application based on the prospect value with multiple reference points," *Inf Sci (N Y)*, vol. 385–386, pp. 415–437, Apr. 2017, doi: 10.1016/j.ins.2017.01.007.

[8] V. A. Ojeda, L. Lázaro, J. J. Benito, and J. M. Bastidas, "Assessment of cathodic protection by close interval survey incorporating the instant off potential method," *Corrosion Engineering, Science and Technology*, vol. 51, no. 4, pp. 241–247, May 2016, doi: 10.1179/1743278215y.0000000041.

[9] S. Wan and J. Dong, "Aggregating Decision Information into Interval-Valued Intuitionistic Fuzzy Numbers for Heterogeneous Multi-attribute Group Decision Making," in *Decision Making Theories and Methods Based on Interval-Valued Intuitionistic Fuzzy Sets*, Springer Singapore, 2020, pp. 139–177. doi: 10.1007/978-981-15-1521-7_5.

[10] D. Khamroeva, P. Kalkhanov, and U. Khamroev, "Algorithm and software for computing the eigenvalue problem of a symmetric interval matrix," in *INTERNATIONAL SCIENTIFIC CONFERENCE ON MODERN PROBLEMS OF APPLIED SCIENCE AND ENGINEERING: MPASE2024*, AIP Publishing, 2024, p. 20018. doi: 10.1063/5.0241528.

[11] T. Sakurai and H. Sugiura, "A projection method for generalized eigenvalue problems using numerical integration," *J Comput Appl Math*, vol. 159, no. 1, pp. 119–128, Oct. 2003, doi: 10.1016/s0377-0427(03)00565-x.

[12] H. Ibn-Khedher, M. Ibn Khedher, and M. Hadji, "Mathematical Programming Approach for Adversarial Attack Modelling," in *Proceedings of the 13th International Conference on Agents and Artificial Intelligence*, SCITEPRESS - Science and Technology Publications, 2021. doi: 10.5220/0010324203430350.

[13] L. E. Sanchis, *Mathematical Semantics for Higher Order Programming Languages.* 1980. doi: 10.21236/ada089733.

[14] Y. Wang and D. Kong, "Jacobian Nonsingularity in Nonlinear Symmetric Conic Programming Problems and Its Application," *Math Probl Eng*, vol. 2020, pp. 1–9, Dec. 2020, doi: 10.1155/2020/8824126.

[15] S. Manzetti, "Mathematical Modeling of Rogue Waves, a Review of Conventional and Emerging Mathematical Methods and Solutions," Apr. 2018, doi: 10.20944/preprints201803.0135.v2.

[16] R. E. Moore, R. B. Kearfott, and M. J. Cloud, *Introduction to Interval Analysis*. Society for Industrial and Applied Mathematics, 2009. doi: 10.1137/1.9780898717716.

[17] K. KH, R. I. M. KH., A. D.A., and A. Z. KH., "MICROBIOLOGICAL CHARACTERISTICS OF THE ORAL MUCOSAIN PATIENTS WITH LICHEN PLANUS ERYTHEMATOSUS," *International Journal of Medical Science and Dental Health*, vol. 10, no. 10, pp. 1–8, Oct. 2024, doi: 10.55640/ijmsdh-10-10-01.