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Singular points classification of first order differential equations system not solved for derivatives

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Abstract

The article considers the singular system points' classification of first-order *differential equations that are not resolved with respect to derivatives. By displaying using some ratios that considered integral curves locations by special point in space taking into account the mutually located integral parabolas we obtain the corresponding pictures of the integral curves location by the singular system point. The types of singular equation points can be classified according the characteristic equation roots type, which we call a saddle, node, focus, center, or saddle focus.*

Keywords: *singular point, classification, image, integral linear surface, equivalent, family, types, knot, saddle, focus, saddle focus, ascending, non-descending, elliptic, parabolic, oscillate, first approximation.*

Introduction

We will consider the singular system points classification of first-order differential equations that are not resolved with respect to derivatives.

$F_k(x,y,z,y',z') = 0 \,, (k = 1,2) \qquad (1)$

where the function F_k has continuous partial derivatives up to the third order inclusive in all variables in some space region (x,y,z,y',z') and we give singular system points classification.
The equations system (1) in the space $(x,y,z,y' = p, z' = q)$ depicts some surface *S*. [1]

Methods and materials:

If the function $y = f(x), z = g(x)$ is a solution to (1) then the equation $p = f'(x), q = g'(x)$ define a curve on the surface *S* for which the relations

$-pdx + dy = 0, -qdx + dz = 0 \qquad (2)$

Conversely, if the curve $x = x(t), y = y(t), z = z(t), p = p(u), q = q(u)$ lies on the surface *S*, it satisfies relations (2), then this curve projection in the space (x,y,z) will be the integral system linear (1). To system (1), we associate each surface point *S* with a linear element элементdx, dy, dz, dp, dq defined by the matrix

$$\begin{Vmatrix} F_{1x} & F_{1y} & F_{1z} & F_{1p} & F_{1q} \\ -p & 1 & 0 & 0 & 0 \\ F_{2x} & F_{2y} & F_{2z} & F_{2p} & F_{2q} \\ -q & 0 & 1 & 0 & 0 \end{Vmatrix}$$

then we can write:

$$\left.\begin{aligned}\frac{dx}{dt} &= F_{1p}F_{2q} - F_{2p}F_{1q} \\ \frac{dy}{dt} &= p(F_{1p}F_{2q} - F_{2p}F_{1q}) \\ \frac{dz}{dt} &= q(F_{1p}F_{2q} - F_{2p}F_{1q}) \\ \frac{dp}{dt} &= F_{1x}F_{2q} - F_{2x}F_{1q} - p(F_{1y}F_{2q} - F_{2y}F_{1q}) + q(F_{1z}F_{2q} - F_{2z}F_{1q}) \\ \frac{dq}{dt} &= F_{1x}F_{2p} - F_{2x}F_{1p} - p(F_{2y}F_{1p} - F_{1y}F_{2p}) + q(F_{1z}F_{2p} - F_{2z}F_{1p})\end{aligned}\right\} \quad (3)$$

If the parameter t is considered temporary, then system (3) determines the point motion law along the line G on the surface *S*.

Definition. If the point $(x_0, y_0, z_0, p_0, q_0)$ of the surface *S* is a singular system point (3), then the point (x_0, y_0, z_0) is called a singular point for system (1).

Let us take the studied singular system point (x_0, y_0, z_0) (1) as the origin. Then $x = y = z = p = q = 0$ is a solution to system (3) and the origin *O* will be a singular system point (1). With the same notation, the Taylor formula for the function F_k will be:

$$\begin{aligned}F_k(x,y,z,p,q) &= F_k(0) + \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z} + p\frac{\partial}{\partial p} + q\frac{\partial}{\partial q}\right) F_k(0) \\ &+ + \frac{1}{2!}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z} + p\frac{\partial}{\partial p} + q\frac{\partial}{\partial q}\right)^2 F_k(0) \\ &+ o(x^3 + y^3 + z^3 + p^3 + q^3); (4)\end{aligned}$$

Since $\lim_{x\to 0}\frac{xp}{y} = 1,\ \lim_{x\to 0}\frac{xq}{z} = 1,\ then\ xp\ and\ y; xq\ и z,$ are equivalent infinitesimal quantities. In addition, taking into account that $F_k(0) = 0,\ F_{kx} = 0, F_{kp} = 0, F_{kq} = 0$, we can write equation (3) as follows:

$$F_{ky}y + F_{kz}z + \frac{1}{2}(F_{kpp})p^2 + (F_{kxp})xp + \frac{1}{2}(F_{kqq})q^2 + (F_{kxq})xq + \ +(F_{kpq})pq + \frac{1}{2}(F_{kxx})x^2 + 0(x^3 + p^3 + q^3) = 0 \quad (5)$$

A system of differential equations

$$F_{ky}y + F_{kz}z + F_{kp}p + F_{kq}q + \frac{1}{2}(F_{kpp})p^2 + (F_{kpq})pq + \frac{1}{2}(F_{kqq})q^2 + +(F_{kxp})xp + (F_{kxq})xq + \frac{1}{2}(F_{kxx})x^2 = 0 \quad (6)$$

Let us call the first approximation of the system (4)

The reason for this name is the fact that the terms $o(x^3 + p^3 + q^3)$ generally do not have any effect on the integral curves behavior near the singular system point (5). Differentiating system (6) with respect to x and considering *p*,*q* as *x* function, we obtain

$$\left.\begin{aligned}\frac{dp}{dx} &= \frac{a_{21}x + a_{12}p + a_{23}q + \varphi_2(x,p,q)}{a_{11}x + a_{12}p + a_{13}q + \varphi_1(x,p,q)} \\ \frac{dq}{dx} &= \frac{a_{31}x + a_{32}p + a_{33}q + \varphi_3(x,p,q)}{a_{11}x + a_{12}p + a_{13}q + \varphi_1(x,p,q)}\end{aligned}\right\} (7),$$

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whereφ_k- polynomials containing terms higher than the first

If, $\varphi_k(x,p,q,c_1,c_2) = 0$, the general system solution (7) then the integral system lines (6) will be the projections in the space *(x,y,z)* of the intersection surface line (6) with the cylindrical surface family $\varphi_k(x,p,q,c_1,c_2) = 0$. We will assume that $\Delta = det\|a_{ij}\| \neq 0$

If, $\Delta = 0$, then each discriminant curve point is singular, and the discriminant curve itself is a singular integral system line (6)

The types of singular equations points (7) can be classified depending on the characteristic equation roots form [3], [4].

$\det(A - \lambda E) = 0$

1. If all roots are valid and of the same sign, then the singular point is a node.
2. If all roots are valid, but have different signs, then the singular point is a saddle.
3. If among the roots there is one real and two complex conjugates. The sum of which has the same sign as the real root, then the singular point is the focus.
4. If among the roots there is one real and two complex conjugates, the sum of which has a sign. Opposite to the real root sign, the singular point is the saddle focal point.
5. If two complex conjugate roots are purely imaginary, then the singular point can be a center, focus, or saddle focus.

By displaying using some ratios that considered integral curves locations near the singular point in the (*x,p,q)*space and taking into account the mutually located integral parabolas we obtain the corresponding pictures of the integral curves location near the singular system point (6). In case 1-3, the integral system lines (7) correspond to three ascending (non-descending) parabolas, which are integral system lines (6). These parabolas pass through the singular point and touch the *OX* axis there. All other integral curves system (7) correspond to integral curves system (6), which are found through a singular point and tangent to the *OX* axis in it. A singular point of this type will be called an elliptical singular point. In cases 2 and 4, the integral system lines (7) correspond to such three parabolas, which are integral system lines (6), but in this case one of the parabolas is ascending and the other two are non-descending. These parabolas touch at a singular point on the *OX* axis. All other system curves (7) correspond to integral system curves (6) of the saddle type.

A singular point of this type will be called a hyperbolic singular point. In case 5, the spirals and closed curves of system (6) correspond to integral curves (6) with cusps on the discriminant parabola.

Integral curves infinitely often oscillate around a singular point, approaching it without a specific direction. A singular point of this type will be called a parabolic singular point.

Conclusion. It can be easily shown that the integral curves location pattern in a sufficiently small origin neighborhood for system (1) is similar to the integral curves location pattern of the first approximation system (6).

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