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Classification of Differential Equations

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Abstract: In the article, the classification of the differential equations of the second order, characterized by the differential equations of the particular derivative, is written in many problems of mathematical physics.

Keywords: special derivative equation, quasi-linear equations, characteristic equation, hyperbolic type equation, elliptic type equation, parabolic type equation, problems characterized by parabolic type equations.

INTRODUCTION. Many problems of mathematical physics are characterized by partial differential equations. The most common among them are differential equations of the second order.

It is a second-order equation with a special derivative depending on two independent variables x, y, and the relationship between the unknown function u(x,y) and its second-order special derivatives

F$(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy})$ =0

is said to In the same way, equations are written for multivariable functions.

MAIN PART. A differential equation is said to be linear with respect to higher-order derivatives if it has the following form

$$a_{11}u_{xx} + 2\,a_{12}u_{xy} + \ a_{22}u_{yy} + F_1\big(x, y, u, u_x, u_y\big) = 0$$

Here a_{11}, a_{12}, a_{22} - coefficients are functions of variables x and y.

If the coefficients a_{11}, a_{12}, a_{22} depend not only on the variables x and y, but also on x, y, u, u_x, u_y l, just like the function F_1, then such equations are called quasi-linear equations.

A differential equation is called linear if it is linear with respect to the higher-order derivatives u_{xx}, u_{xy}, u_{yy} and the function $u(x, y)$ and its first-order derivatives u_x, u_y ;

$a_{11}u_{xx} + 2\,a_{12}u_{xy} + \ a_{22}u_{yy} + b_1u_x + b_2u_y + cu + f = 0$ Here $a_{11},\ a_{12},\ a_{22},\ b_1, b_2, c, f,$ are only functions of variables x, y. If the coefficients of equation (2) do not depend on the variables x and y, then this equation consists of a linear equation with constant coefficients. The equation is called homogeneous if $f(x, y) = 0$. The above equations use these designations.

$u_x = \frac{du}{dx}, u_y = \frac{du}{dy}, u_{xx} = \frac{d^2u}{dx^2}, u_{xy} = \frac{d^2u}{dxdy}, u_{yy} = \frac{d^2u}{dy^2},$ etc.

As a result of changing the variables as follows

$\xi = \rho(x, y) \quad\ \eta = \varphi(x, y)$

We get a new equation that is equivalent to the original equation. Such a question arises, that is, how to choose the variables ξ and η, so that the equations written by these variables have a simple appearance. We answer this question by the following equation, which is linear with respect to higher order derivatives and depends on two variables x, y

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$a_{11}u_{xx} + 2\, a_{12}u_{xy} + \ a_{22}u_{yy} + F(x, y, u, u_x, u_y) = 0$

Let's look at the example. Expressing the variables in terms of new variables, we form these equations:

$u_x = u_\xi \xi_x + u_\eta \eta_x$,

$u_y = u_\xi \xi_y + u_\eta \eta_y$,

$u_{xx} = u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\xi \xi_{xx} + u_\eta \eta_{xx}$, (3)

$u_{xy} = u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + u_{\eta\eta}\eta_x\eta_y + u_\xi \xi_{xy} + u_\eta \eta_{xy}$,

$u_{yy} = u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}\eta_y^2 + u_\xi \xi_{yy} + u_\eta \eta_{yy}$.

If we put these values of derivatives from relations (3) into (1), we get:

$\bar{a}_{11}u_{\xi\xi} + 2\bar{a}_{12}u_{\xi\eta} + \bar{a}_{22}u_{\eta\eta} + \bar{F} = 0$,

Here

$\bar{a}_{11} = a_{11}\xi_x^2 + 2a_{12}\xi_x\xi_y + a_{22}\xi_y^2$,

$\bar{a}_{12} = a_{11}\xi_x\eta_x + a_{12}(\xi_x\eta_y + \eta_x\xi_y) + a_{22}\xi_x\eta_y$,

$\bar{a}_{22} = a_{11}\eta_x^2 + 2a_{12}\eta_x\eta_y + a_{22}\eta_y^2$.

Functions F¯ do not depend on second-order derivatives. It should be noted that if the initial equation is linear, i.e

$F(x, y, u, u_x u_y) = b_1u_x + b_2u_y + CU + f$,

then F will be in this form.

$\bar{F}(\xi, \eta, u, u_\xi u_\eta) = \beta_1 u_\xi + \beta_2 u_\eta + JU + f$, (4)

That is, the equation remains linear. Now we choose the variables ξ and η so that the coefficient $\bar{a}_{11}$ is equal to zero. Consider the following equation with the first order partial derivative.

$a_{11}z_x^2 + 2a_{12}z_xz_y + a_{22}z_y^2 = 0$ (5)

Let us assume that $z = \varphi(x, y)$ is a particular solution of this equation. If we take $\xi = \varphi(x, y)$, then the coefficient $\bar{a}_{11}$ is known to be equal to zero. Thus, the above-mentioned issue of choosing new variables is related to the solution of equation (5).

1. If $z = \varphi(x, y)$ is a function

$a_{11}z_x^2 + 2a_{12}z_xz_y + a_{22}z_y^2 = 0$ (6)

If the equation has a particular solution, then the relation $\varphi(x, y) = C$ is the general integral of this ordinary differential equation.

2. If the relation $\varphi(x, y) = C$ is the general integral of the following ordinary differential equation, then the function $\varphi(x, y) = C$ satisfies equation (5).

We prove the first lemma.

The function $z = \varphi(x, y)$ satisfies equation (5) according to the condition of the lemma, then the equality $a_{11}(\frac{\varphi_x}{\varphi_y})^2$-$2a_{12}\left(-\frac{\varphi_x}{\varphi_y}\right) + a_{22} = 0$ (7)

it will consist of an item. This equation is satisfied in the domain where the solution is given for all x, y. The relation $\varphi(x, y) = C$, is the general integral of equation (6) if the function y determined from the implicit relation $\varphi(x, y) = C$ satisfies equation (6).

Suppose the desired function

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$y = \mathrm{f(x, c)}$; in that case

$$\frac{dy}{dx} = -\left[\frac{\varphi_x(x,y)}{\varphi_y(x,y)}\right]_{y=f(x,c)} \quad (8)$$

will be, where the square brackets and the symbol $y = \mathrm{f(x, c)}$ indicate that the variable y on the right side of equation (8) is not a free variable and takes a value equal to $y = \mathrm{f(x, c)}$ shows.

It follows that $y = \mathrm{f(x, c)}$ satisfies equation (6) because

$$a_{11}(\frac{\varphi_x}{\varphi_y})^2\text{-}2a_{12}\left(-\frac{\varphi_x}{\varphi_y}\right) + a_{22} = \left[a_{11}(-\frac{\varphi_x}{\varphi_y})^2 - 2a_{12}\left(-\frac{\varphi_x}{\varphi_y}\right) + a_{22}\right]_{y=f(x,c)} = 0,$$

the expression in the square brackets is equal to zero at all values of x, y, when the square is $y = f(x, c)$. The lemma is proved.

Now we prove the second lemma.

Let us assume that $\mathrm{\varphi(x, y) = C}$ is a function, the general integral of equation (6). For an arbitrary point (x, y).

$$a_{11}\varphi_x^2 + 2a_{12}\varphi_x\varphi_y + a_{22}\varphi_y^2 = 0'$$

we will prove it. Now let (x_0, y_0) be any given point. If we prove that the equality $((7))'$ is satisfied for this point, then since the point (x_0, y_0) is an arbitrary point, the equality $((7))'$ is real, and the function φ(x,y) is the equation $((7))'$ turns out to be a solution of.

Now we pass the integral curve of equation (6) through the point (x_0, y_0) and consider that $\varphi(x_0, y_0) = C_0$ and look at the curve $y = f(x, C_0)$. It is known that $y_0 = f(x_0, C_0)$. For all points of this curve we have:

$$a_{11}\left(\frac{dy}{dx}\right)^2 - 2a_{12}\frac{dy}{dx} + a_{22} = \left[a_{11}\left(-\frac{\varphi_x}{\varphi_y}\right)^2 - 2a_{12}\left(-\frac{\varphi_x}{\varphi_y}\right) + a_{22}\right]_{y=f(x,c)} = 0.$$

If we take $x = x_0$ in the last equation, we get the required property

$$a_{11}\,\varphi_x^2(x_0y_0) + 2a_{12}\varphi_x(x_0y_0)\varphi_y(x_0y_0) + a_{22}\varphi_y^2(x_0y_0) = 0\ ,$$

Equation (6) is called the characteristic equation for equation (1), and its integrals are called characteristics.

Equation (6) splits into two equations:

$$\frac{dy}{dx} = \frac{a_{12} + \sqrt{a_{12}^2 - a_{11}a_{22}}}{a_{11}}, (9)$$

$$\frac{dy}{dx} = \frac{a_{12} - \sqrt{a_{12}^2 - a_{11}a_{22}}}{a_{11}}, (10)$$

In this case, the sign of the expression in the root of this equation

$$a_{11}u_{xx} + 2\,a_{12}u_{xy} + \ a_{22}u_{yy} + F = 0$$

determines the type.

We call this equation a hyperbolic type equation at the point M, if $a_{12}^2 - a_{11}a_{22} > 0$, we call it an elliptic type equation, if $a_{12}^2 - a_{11}a_{22} < 0$ to the point M, and a parabolic type equation, if M to the point $a_{12}^2 - a_{11}a_{22} = 0$.

Now let's consider the processes described by differential equations with particular derivatives on the example of parabolic type equations. In this case, equations of the second-order private derivative parabolic type are often found in the study of heat conduction and diffusion processes. One of the simplest parabolic equations is the heat transfer equation:

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$$\frac{du}{dt} - \frac{d^2u}{dx^2} = 0 .$$

Let's look at problems described by equations of parabolic type

1. The linear problem of heat dissipation. Let's look at a rod whose length is equal to L, we consider it protected from heat from the sides and thin enough, which allows us to consider the temperature in the cross section of the rod at an arbitrary moment of time to be the same.

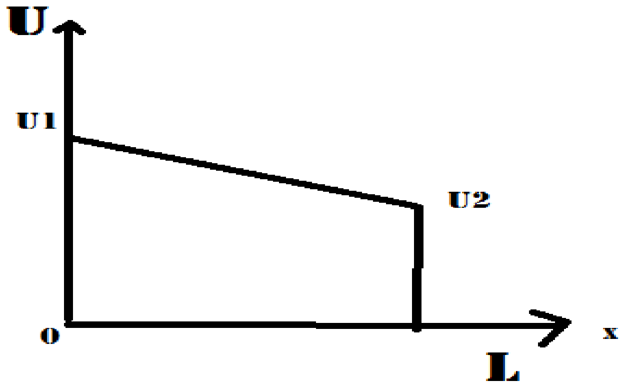

Figure 1

If we keep the extreme points of the stern at constant temperatures u_1 and u_2, as is known, a linear distribution of temperature along the stern is established (Fig. 1).

$u(x) = u_1 + \frac{u_2-u_1}{L}x.$ (11)

In this case, heat flows from the extreme points of high movement of the rod to the extreme point of low movement. The amount of heat that flows through the surface of the cross-section S of the stern during a unit time is given by this experimental formula.

$Q = -k\frac{U_1-U_2}{L}\ S = -k\frac{du}{dx}S$ (12)

Here, $k -$ is the heat transfer coefficient, which depends on the material of the rod. The amount of heat flow is considered positive if the heat flows in the direction of increasing x.

Let's look at the process of heat dissipation in the stern. This process is described by a two-argument function $u(x,t)$, which represents the temperature at point x of the stern at time t. We derive the equation that satisfies the function $u(x,t)$.

Foure's law. If the temperature of the object is different, then the heat flow will move from the place of higher temperature to the place of lower temperature.

The amount of heat flowing through section x in the interval of time $(t, t + dt)$ is equal to the following.

$dQ = qSdt,$ (13)

Here

$q = -k(x)\frac{du}{dx}$ (14)

is the heat flow density, i.e. it is equal to the amount of heat flowing through the surface of cm^2 in a unit time interval. This law is a generalization of formula (12). This formula can also be given an integral form.

$Q = -S\int_{t_1}^{t_2} k\frac{du}{dx}(x,t)dt,$ (15)

Here, Q- x is the amount of heat flowing through section x during the time interval (t_1, t_2). If the sturgeon is not homogeneous, then the coefficient k is a function of x.

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2. To increase the temperature of a homogeneous body by $\triangle$u, the amount of heat supplied to it is equal to the following.

$Q = cm \triangle u = c\rho V \triangle u,$ (16)

Here, c –is the relative heat capacity, m – is the mass of the object, ρ- is its density, and V- is the volume.

If the temperature change is different at different points of the stern, or the stern is not homogeneous, then the heat flow is equal to the following.

$Q = \int_{x_1}^{x_2} c\rho S \triangle u(x)dx.$ (17)

3. Heat can be generated or absorbed inside the vessel (for example, when a current is passed, as a result of a chemical reaction, etc.). Heat dissipation can be characterized by the density function of heat sources $F(x,t)$, which consists of the source at point x of the stern at time t. As a result of the influence of these sources, the following amount of heat is released in the $(x, x + \triangle x)$ part of the stern at the moment of time $((t, t + dt)$.

$dQ = SF(x,t)dx\, dt$ (18)

Or in integral form

$Q = S\int_{t_1}^{t_2}\int_{x_1}^{x_2} F\,(x,t)dx\, dt,$ (19)

will be, where Q means the amount of heat released in the (x_1, x_2) part of the stern during the time interval (t_1, t_2). The heat transfer equation is formed by expressing the heat balance at the moment of time (t_1, t_2) in a section (x_1, x_2). Using the law of conservation of energy, we can write the equation below using formulas (15), (17) and (19).

$\int_{t_1}^{2_2}\left[k\frac{du}{dx}(x,\tau)\Big|_{x=x_2} - k\frac{du}{dx}(x,\tau)\Big|_{x=x_2}\right]d\tau + \int_{x_1}^{x_2}\int_{t_1}^{t_2} F(\xi,\tau)d\xi\, d\tau = = \int_{x_1}^{x_2} c\rho[u(\xi,t_2) - u(\xi,t_1)]d\xi,$ (20)

This formula consists of writing the heat transfer equation in the integral form.

To write the heat transfer equation in the differential form, we assume that the function $u(x,t)$ has the following continuous eigenderivatives, i.e.

$U_{xx} = \frac{d^2u}{dx^2}$ and

$$U_t = \frac{du}{dt}$$

then, if we use the mean value theorem, we will have the following equality.

$\left[k\frac{du}{dx}\;(x,\tau)\;\Big|_{x=x_2} - k\frac{du}{dx}(x,\tau)\;\Big|_{x=x_1}\right]\tau{=}t_3$

$\triangle t = F(x_4, t_4) \triangle x \triangle t = \{c\rho[u(\xi,t_2) - u(\xi,t_1)]\}_{\xi=x_3} \triangle x,$ (21)

This equation can be written as follows using the finite sum theorem.

$$\frac{d}{dx}\left[k\frac{du}{dx}(x,t)\right]_{\substack{x=x_5\\ t=t_5}} \triangle t \triangle x + F(x_4,t_4) \triangle x \triangle t$$

$= \left[c\rho\frac{du}{dt}(x,t)\right]_{\substack{x=x_3\\ t=t_5}} \triangle x \triangle t,$ (22)

Here, points t_3, t_4, t_5 and x_3, x_4, x_5 are intermediate points of intervals (t_1, t_2) and $x_1, x_2)$, respectively. As a result of reducing the last equation to $\triangle$x$\triangle$t, we get this.

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$$\frac{d}{dx}\left(k\frac{du}{dx}\right)\Big|_{\substack{x=x_5\\t=t_5}}+F(x,t)\Big|_{\substack{x=x_4\\t=t_4}}=c\rho\frac{du}{dt}\Big|_{\substack{t=t_5\\x=x_3}}. \quad (23)$$

CONCLUSION

All the above considerations are valid for arbitrary (t_1, t_2) and (x_1, x_2) intervals. If we move from this equation to the limits $x_1, x_2 \to x_3$and $t_1, t_2 \to t_3$, we will create the heat transfer equation.

$$\frac{d}{dx}\left(k\frac{du}{dx}\right)+F(x,t)=c\rho\frac{du}{dt}. \quad (24)$$

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