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METHODS OF SOLVING MODULAR INEQUALITIES

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Annotation
In the article, in the usual method for solving problems related to modular inequalities and their applications, based on the definition of the modulus of a real number, the set of values of the unknown is divided into disjoint parts, each of which retains its sign and solves the set of inequalities. A set of methods for solving inequalities is listed, and in some cases it is explained that other methods can be used. Modular inequalities, along with solving problems related to their applications in real life, are considered one of the important directions of the subject. Addressing problems directly related to the scientific and practical challenges encountered in daily life is a crucial aspect of the comprehensive education system at all levels. This application holds practical significance across all stages of the education system.
Key words: *modular inequalities, modulus of a real number, function, the set of inequalities, method.*

The use of non-traditional methods in addition to the traditional methods, effective use of new pedagogical technologies, computer and information technologies, natural and social One of the urgent issues is the use of new methods in teaching subjects, including mathematical analysis, differential equations, mathematical physics equations, solving important practical problems, and issues of development and improvement of such methods.

In order to organize the educational process according to the purpose and to eliminate the noted shortcomings in the pedagogical activity of teachers and their professional training:

- humanizing and democratizing relations between teachers and students;
- formation of conscious discipline by organizing and managing students' cognitive activities in the educational process;
- to achieve educational efficiency through the harmonious use of modern pedagogical and information technologies in teaching;
- formation of independent and creative activities by organizing students' cognitive activities on the basis of independent and creative research;

it is necessary to develop and put into practice the scientific-methodical foundations of the differentiated approach to the educational process, taking into account the interests, needs and abilities of students.

In mathematics itself, there are separate topics and issues related to the indicated issues. One of them is modular inequalities and examples of their applications. Modular inequalities, issues of their application and their solution is a science direction aimed at solving scientific and practical problems encountered in real life. Therefore, a systematic analysis of the teaching of inequalities

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and their application at all stages of education, that is, in general schools, academic lyceums and vocational colleges, and in higher education, is one of the most urgent issues.

In the usual method of solving inequalities involving the modulus sign, the set of values of the unknown is divided into non-intersecting parts, each of which retains its sign, based on the definition of the modulus of a real number, and proceeds to solving the set of inequalities.

This is the basic method, and you have to deal with a lot of cases when solving some examples. Therefore, in some cases, other methods can be used.

Rule 1: $|f(x)| \le g(x) \Leftrightarrow \begin{cases} f(x) \le g(x) \\ f(x) \ge -g(x) \end{cases} \quad (*)$

$|f(x) \ge g(x)| \Leftrightarrow \begin{cases} f(x) \ge g(x) \\ f(x) \le -g(x) \end{cases} \quad (**)$

the equal strength property can be used [3;134].

Rule 2: $|f(x)| > |g(x)| \Leftrightarrow [f(x)]^2 > [g(x)]^2 \quad (***)$

$|f(x)| < |g(x)| \Leftrightarrow [f(x)]^2 < [g(x)]^2 \; (****)$

Rule 3: $|a| \prec b \Leftrightarrow -b \prec a \prec b$ equal strength is also used.

Rule 4: Some simple inequalities $\rho(M,N)$ can also be solved using the concept of distance.

Example 1: $|x-2|<3$ solve the inequality.

a) We use rule 3.

$|x-2|<3 \Leftrightarrow -3<x-2<3 \Rightarrow -1<x<5$

b) $|x-2| = p\,(M(x), N(2)) < 3,$ the distances to these 2 points are less than 3^{x} set of points: $-1<x<5$

v) Rule 2 above $(****)$:

$|x-2|<3 \Leftrightarrow \begin{cases} x-2<3 \\ x-2>-3 \end{cases} \Rightarrow \begin{cases} x<5 \\ x>-1 \end{cases} \Rightarrow -1<x<5$

d) According to the general rule:

$$|x-2|<3 \Leftrightarrow \left[\begin{array}{l} \begin{cases} x<2 \\ -x+2<3 \end{cases} \\ \begin{cases} x\ge 2 \\ x-2<3 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} x<2 \\ x>-1 \end{cases} \\ \begin{cases} x\ge 2 \\ x<5 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} -1<x<2 \\ 2\le x<5 \end{array}\right. \Leftrightarrow -1<x<5$$

Example 2: $|1-3x|>2$ solve the inequality.

$$|1-3x|>2 \Leftrightarrow \left[\begin{array}{l} 1-3x>2 \\ 1-3x<-2 \end{array}\right. \Rightarrow \begin{cases} -3x>1 \\ -3x<3 \end{cases} \Leftrightarrow \left[\begin{array}{l} x<-\frac{1}{3} \\ x>1 \end{array}\right. ,\; x<-\frac{1}{3} \;,\; x>1$$

Example 3: $|2x-1| \le |3x+1|$ solve the inequality.

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$(2x-1)^2 \le (3x+1)^2 \Leftrightarrow 4x^2-4x+1 \le 9x^2+6x+1 \Leftrightarrow 5x^2+10x \ge 0 \Leftrightarrow 5x(x+2 \ge 0)$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} x \ge 0 \\ x+2 \ge 0 \end{cases} \\ \begin{cases} x \le 0 \\ x+2 \le 0 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x \ge 0 \\ x \le -2 \end{array}\right.$$

So, $x \le -2$ *va* $x \ge 0$ a set of solutions.

Example 4: $\left|\frac{3x+1}{2x-1}\right| > 1$ solve the inequality.

$2x-1 \ne 0 \Rightarrow \quad x \ne \frac{1}{2} \ \rightarrow \ \left|\frac{3x+1}{2x-1}\right| = \frac{|3x+1|}{|2x-1|}$ that is why $\frac{|3x+1|}{|2x-1|} > 1$ we solve the inequality.

$\frac{|3x+1|}{|2x-1|} > 1 \Leftrightarrow |3x+1| > |2x-1| \Leftrightarrow (3x+1)^2 > (2x+1)^2$ ago

$9x^2+6x+1 > 4x^2-4x+1$ *yoki* $5x^2+10x > 0$ became a quadratic equation. $5x(x+2) > 0$ set of solutions to the inequality $x < -2$ and $x > 0$ consists of but above $x \ne \frac{1}{2}$ we had said. Hence the solution set $(-\infty : -2) \cup \left(0 : \frac{1}{2}\right) \cup \left(\frac{1}{2} : +\infty\right)$ will consist of

Example 5: $|x-1| + |2x+3| > 7$ solve the inequality [1; 67].

Above $(****)$ we apply the rule::

$$\left[\begin{array}{l} \left[\begin{array}{l} 2x+3 > 8-x \\ 2x+3 < x-8 \end{array}\right. \\ \left[\begin{array}{l} 2x+3 > x+6 \\ 2x+3 < -x-6 \end{array}\right. \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \left[\begin{array}{l} x > \frac{5}{3} \\ x < -11 \end{array}\right. \\ \left[\begin{array}{l} x > 3 \\ x < -3 \end{array}\right. \end{array}\right.$$

$$|x-1| + |2x+3| > 7 \Leftrightarrow |x-1| > 7 - |2x+3| \Leftrightarrow \left[\begin{array}{l} x-1 > 7-|2x+3| \\ x-1 < |2x+3|-7 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} |2x+3| > 8-x \\ |2x+3| > x+6 \end{array}\right. \Leftrightarrow$$

$x < -11$ $\qquad x > 3$

So, $(-\infty : -11) \cup (3 : \infty)$ will be a set of solutions to the inequality.

Example 6: $|x-1| + |x-2| < x+3$ solve the inequality.

$|x-1| + |x-2| < x+3 \Leftrightarrow |x-1| < x+3-|x-2| \Leftrightarrow$

$$\begin{cases} x-1 < x+3-|x-2| \\ x-1 > |x-2|-x-3 \end{cases} \Leftrightarrow \begin{cases} |x-2| < 4 \\ |x-2| < 2x+2 \end{cases} \Leftrightarrow \begin{cases} -4 < -2 < 4 \\ \begin{cases} x-2 < 2x+2 \\ x-2 > -2x-2 \end{cases} \end{cases} \Leftrightarrow \begin{cases} -2 < x < 6 \\ \begin{cases} x > -4 \\ x > 0 \end{cases} \end{cases} \Leftrightarrow \begin{cases} -2 < x < 6 \\ x > 0 \\ 0 < x < 6 \end{cases} \Leftrightarrow$$

So, $(0 : 6)$ set of solutions to the inequality.

Example 7: $|x| + |x-1| + |x-2| < 4$ solve the inequality.

We solve this inequality by applying the general rule.

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We find the points that make each term zero, and divide the intervals where the signs of the terms do not change.

These are the intervals $(-\infty:0),[0:1),[1:2),(2:\infty)$ will be.

1) $x\in(-\infty:0)$ $\quad x<0, x-1<0, x-2<0$ will be. That is why $|x|=-x, |x-1|=1-x, |2-x|=x-2$ will be.

2) $x\in[0:1)$, $x.0, x-1<0,\ x-2<0$ being,

$|x|=x, |x-1|=1-x, |x-2|=2-x$ will be.

3) $x\varepsilon[1:2)$, $x>0, x-1>0, x-2<0$ being, $|x|=x, |x-1|=1-x, |x-2|=2-x$ will be.

4) $x\in(2:\infty)$, $x>0,\ x-1>0,\ x-2>0$ being, $|x|=x,\ |x-1|=x-1,\ |x-2|=x-2$ will be.

Thus, the given inequality is as strong as the following set of systems.

$$\left[\begin{array}{l}\begin{cases}x<0\\-3x<1\end{cases}\\ \begin{cases}0\le x<1\\-x<1\end{cases}\\ \begin{cases}1\le x<2\\x<3\end{cases}\\ \begin{cases}x\ge 2\\3x<7\end{cases}\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\begin{cases}x<0\\x>-\frac{1}{3}\end{cases}\\ \begin{cases}0\le x<1\\x>-1\end{cases}\\ \begin{cases}1\le x<2\\x<3\end{cases}\\ \begin{cases}x\ge 2\\x<\frac{7}{3}\end{cases}\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\begin{cases}-\frac{1}{3}<x<0\\0\le x<1\end{cases}\\ 1\le x<2\\ 2\le x<\frac{7}{3}\end{array}\right.\Leftrightarrow -\frac{1}{3}<x<\frac{7}{3}$$

So, $\left(-\frac{1}{3}:\frac{7}{3}\right)$ given inequalities will be solutions of equations.

Example 8: $\left|\frac{x^2-3x+2}{x^2+3x+2}\right|>1$ solve the inequality.

$$\left|\frac{x^2-3x+2}{x^2+3x+2}\right|>1\Leftrightarrow|x^2-3x+2|>|x^2+3x+2|,\qquad (x\ne-2,\ x\ne-1)$$

This inequality can be satisfied using rules 1 and 2 in this paragraph:

$$|x^2-3x+2|>|x^2+3x+2|\Leftrightarrow\left[\begin{array}{l}x^2-3x+2>|x^2+3x+2|\\x^2+3x+2<-|x^2+3x+2|\end{array}\right.\Leftrightarrow$$

$$\left[\begin{array}{l}|x^2+3x+2|<x^2-3x+2\\|x^2+3x+2|<-x^2-3x+2\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\begin{cases}x^2+3x+2<x^2-3x-2\\x^2+3x+2>-x^2+3x-2\end{cases}\\ \begin{cases}x^2+3x+2<-x^2+3x-2\\x^2+3x+2>x^2-3x+2\end{cases}\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\begin{cases}x<0\\x^2>-2\end{cases}\\ \begin{cases}x^2<-2\\x>0\end{cases}\end{array}\right.\Rightarrow\left[\begin{array}{l}x<0\\ \otimes\end{array}\right.\Rightarrow x<0$$

A set of inequality solutions $(-\infty:-2)\cup(-2:-1)\cup(-1:0)$ will be.

Example 9: $|\log_{\sqrt{2}}x-2|-|2-\log_2|>1$ solve the inequality.

We also solve this inequality with one of the previously considered ones, but for the sake of brevity $\log_{a^n}b=\frac{1}{n}\log_a b$ it is possible to use the verb.

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$$\left|\log_{\sqrt{2}} x-2\right|>1+\left|2-\log_2 x\right| \Leftrightarrow \left[\begin{array}{l}\log_2 x^2-2>1+\left|2-\log_2 x\right| \\ \log_2 x^2-2<-1-\left|2-\log_2 x\right|\end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{l}\left|2-\log_2 x\right|<\log_2 x^2-3 \\ \left|2-\log_2 x\right|<1-\log_2 x^2\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\left\{\begin{array}{l}2-\log_2 x<\log x^2-3 \\ 2-\log_2 x>3^2\log_2 x^2\end{array}\right. \\ \left\{\begin{array}{l}2-\log_2 x<1-\log_2 x \\ 2-\log_2 x>\log_2 x^2-1\end{array}\right.\end{array}\right. \Rightarrow$$

$$\left[\begin{array}{l}\left\{\begin{array}{l}\log_2 x^3>5 \\ \log_2 x>1\end{array}\right. \\ \left\{\begin{array}{l}\log_2 x<-1 \\ \log_2 x^3<3\end{array}\right.\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\left\{\begin{array}{l}3\log_2 x>5 \\ \log_2 x>1\end{array}\right. \\ \left\{\begin{array}{l}\log_2 x<-1 \\ 3\log_2 x<3\end{array}\right.\end{array}\right. \Leftrightarrow \left\{\begin{array}{l}\left\{\begin{array}{l}\log_2 x>\frac{5}{3} \\ \log_2 x>1\end{array}\right. \\ \left\{\begin{array}{l}\log_2 x<-1 \\ \log_2 x<1\end{array}\right.\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\left\{\begin{array}{l}x>2\sqrt[3]{4} \\ x>2\end{array}\right. \Rightarrow x>2\sqrt[3]{4} \\ \left\{\begin{array}{l}0<x<0,5 \\ 0<x<2\end{array}\right. \Rightarrow 0<x<\frac{1}{2}\end{array}\right.$$

Example 10: $|x+1|^{x^2-\frac{5x}{2}+\frac{3}{2}}<1 \quad (*)$ we solve this inequality.

$$\left[\begin{array}{l}\left\{\begin{array}{l}0<|x+1|<1 \\ x^2-\frac{5}{2}x+\frac{3}{2}>0\end{array}\right. \\ \left\{\begin{array}{l}|x+1|>1 \\ x^2-\frac{5}{2}x+\frac{3}{2}<0\end{array}\right.\end{array}\right. \left[\begin{array}{l}\left\{\begin{array}{l}-2<x<-1 \\ -1<x<0 \\ \left[\begin{array}{l}x<1 \\ x>\frac{3}{2}\end{array}\right.\end{array}\right. \\ \left\{\begin{array}{l}x>0 \\ 1<x<\frac{3}{2}\end{array}\right.\end{array}\right. \Rightarrow \left[\begin{array}{l}-2<x<-1 \\ -1<x<0 \\ 1<x<\frac{3}{2}\end{array}\right.$$

So the solution set is: $(-2:-1)\cup(-1:0)\cup\left(1:\frac{3}{2}\right)$ will be

Solving the system of inequalities involving the module goes to writing the terms involving the module in the expressions without the module, and then the usual system is solved [4; 94].

Example 11: $\left\{\begin{array}{l}\left|\log_a x\right|<1 \\ \frac{\log_a x}{1-\log_a x}<1\end{array}\right. \quad (a>1)$ solve the system [2;127].

$$\left\{\begin{array}{l}\left|\log_a x\right|<1 \\ \frac{\log_a x}{1-\log_a x}<1\end{array}\right. \Leftrightarrow \left\{\begin{array}{l}\log_a x>-1 \\ \log_a x<1 \\ \log_a x<1-\log_a x\end{array}\right. \Leftrightarrow \left\{\begin{array}{l}\log_a x>-1 \\ 2\log_a x<1\end{array}\right. \Leftrightarrow -1\prec\log_a x<\frac{1}{2}\Rightarrow\frac{1}{a}<x<\sqrt{a}$$

Example 12: $\left\{\begin{array}{l}x+y-7<0 \\ |x-y|<2 \\ 4x>5\end{array}\right.$ remove the system.

$|x-y|>2 \Leftrightarrow -2<x-y<2$ given system because

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$$\begin{cases} x+y-7<0 \\ x-y>-2 \\ x-y<2 \\ 4x>5 \end{cases}$$ is as strong as the system.

Now $x+y=7$, $x-y=-2$, $x-y=2$, $4x=5$ straight lines are drawn in the coordinate system, and then the half-plane representing each inequality is determined.

Hatched area: $\begin{cases} \frac{5}{4}<x<\frac{5}{2} \\ x-2<y<x+2 \end{cases}$ $\begin{cases} \frac{5}{2}<x<\frac{9}{2} \\ x-2<y<-x+7 \end{cases}$ is limited to

So, the set of solutions of the system consists of the face of the hatched area in the plane, only straight lines are not included.

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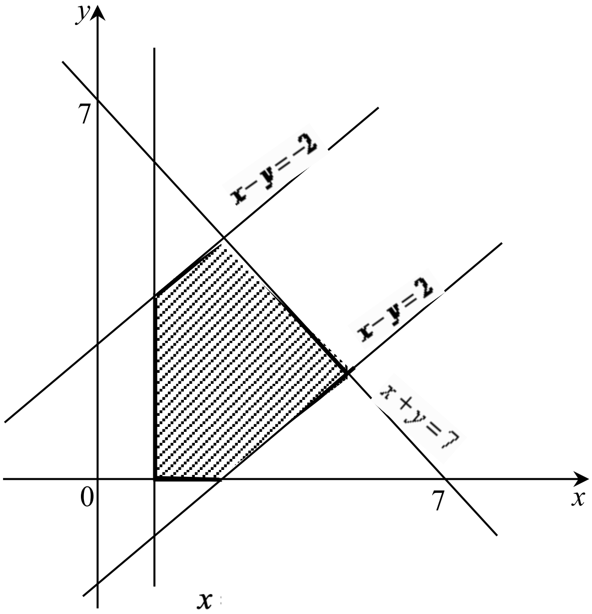

1– picture.

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If we explain how to solve examples to pupils, students, applicants, it will be easier for them to understand. It is more convenient to solve such methods by applying them to the examples in which the rest of the module is involved.

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