

## ABOUT ALGEBRAIC STRUCTURES AND THEIR RANKS

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**Annotation:** Concepts of group, ring and field, linear space, concepts of algebra and color of algebra were presented, rules, ideas were reviewed.

**Key word:** Group, ring, field, linear space, algebra, color of algebra, associative, commutative, distributive, neutral element.

Algebra and numbers theory in the chair group, ring and field concepts determined they are \_ that's it of the subject main concepts is considered

forms a ring with respect to the operations  $+$  and  $\cdot$ , there is a unit element with respect to the operation, and for any element of the ring  $K$  different from  $0$  there is an inverse element, then  $K$  is called a body. So, for the set  $K$  to be a body, the following conditions must be met:

1.  $+$  action associative;
2.  $+$  operation is commutative;
3. a neutral element to the action of  $+$   $\theta$ ;
4. Any element of  $K$  For  $a$ , there is  $-a \in K$ , which is the inverse (opposite) of the action  $+$  ;
5.  $\cdot$  action is associative;
6.  $\cdot$  amali  $+$  distributive relative to amali;
7. In the ring  $K$  there is an element  $e$  such that the equality  $e \cdot a = a \cdot e = a$  is fulfilled;
8. any non-0 element  $a$  in the ring  $K$ , there is an inverse element  $a^{-1} \in K$ , i.e  $a \cdot a^{-1} = a^{-1} \cdot a = e$  equality is fulfilled.

If  $K$  is a body, the following additional condition

9.  $\cdot$  action is commutative;

, then  $K$  is called a field. So the commutative body is a field. Depending on the number of elements in a group, ring, body and field, it is called finite or infinite.

$ax=b$  for any elements  $a$  and  $b$  of the field ( $a \neq 0$ ); There are solutions to the equation  $x \cdot a=b$  and these solutions are the same. But if these solutions exist for body elements  $a$  and  $b$ , they may not be equal.

The concept of linear space is also defined as in the course of algebra and number theory.

If  $L$  the set forms a linear space over the field  $L_1P$ , and the set itself  $\cdot$  forms a ring with respect to the operations  $+$  and  $\cdot$ , and for any  $\lambda \in P$ ,  $x, y \in L$  elements  $(\lambda \circ x) \cdot y = x \cdot (\lambda \circ y)$  (1) are fulfilled, then  $L$  the set is said to form an algebra over the field  $P$ . When writing the equations (1),  $L$  the multiplication operation in the ring was denoted  $\cdot$  by the symbol  $L$  of the operation of multiplying  $\circ$  the vector  $x$  of the linear space  $\lambda \in P$  by the scalar  $\cdot$ .

If a set  $L$  forms an algebra over a field  $P$ ,  $L$  the dimension of the linear space is called the color of the algebra.

If a set forms a group (or a ring, a field, a linear space, an algebra) and its partial set  $L_1$  also  $L$  forms a group (or a ring, a field, a linear space, an algebra), then the  $L_1$  partial  $L$  group (a ring, body, area, space, algebra). This case is also called an extension of  $L$  ni  $L_1$ .

**Examples.**

1. Show the grouping of the set of 6th degree complex roots of 1 with respect to the multiplication operation.

**Solution:**  $C_6 = \{ \alpha \in C \mid \alpha^6 = 1 \}$

$S_6$  be the set of all complex roots of degree 6 of 1. The multiplication operation on this set is defined because

$$\alpha \in S_6, \beta \in S_6, \alpha^6 = 1, \beta^6 = 1$$

the equalities are appropriate, from which the fulfillment of the equality  $(\alpha \cdot \beta)^6 = 1$  also follows. The associativity of this action is fulfilled. 1 being  $\in S_6$ , this element is the unit element with respect to its operation in  $S_6$ . We show that any element in  $S_6$  has an inverse.

$$\left(\frac{1}{\alpha}\right)^6 = \frac{1}{\alpha^6} = 1 \text{ also follows from the equality that } \frac{1}{\alpha} \in S_6 \text{ belongs to the set. So, the set } S_6 \text{ is a finite group}$$

with respect to the operation of multiplication (because there are 6 elements in the set  $S_6$ ).

2.  $Q(\sqrt[3]{3}) = \{a + b\sqrt[3]{3} + c\sqrt[3]{9} \mid a, b, c \in Q\}$  show that the set forms an algebra over the field of rational numbers with respect to simple operations, and determine the color of this algebra.

**Solution :**  $\alpha = a + b\sqrt[3]{3} + c\sqrt[3]{9} \in Q(\sqrt[3]{3})$

$$\beta = a_1 + b_1\sqrt[3]{3} + c_1\sqrt[3]{9} \in Q(\sqrt[3]{3})$$

given the elements,  $\alpha + \beta = (a + a_1) + (b + b_1)\sqrt[3]{3} + (c + c_1)\sqrt[3]{9}$  adding them with equality and  $\lambda \in$  multiplying a vector with equality  $\alpha$  for a number  $\lambda \cdot \alpha = \lambda a + \lambda b\sqrt[3]{3} + \lambda c\sqrt[3]{9} \in Q$  are simple operations. With respect to these operations,  $Q(\sqrt[3]{3})$  the set  $Q$  -constitutes a linear space over the field of rational numbers (linear space axioms are fulfilled). The product of any two  $Q(\sqrt[3]{3})$  and  $\beta$  elements from the set is determined by the rule of multiplication of polynomials.  $\alpha \cdot \beta = (a a_1 + 3(b_1 c + c_1 b) + (a_1 b + a b_1 + 3 c c_1) \sqrt[3]{3} + (a c_1 + a_1 c + b b_1) \sqrt[3]{9}$  will result  $Q(\sqrt[3]{3})$ .  $\alpha \cdot \beta \in Q(\sqrt[3]{3})$  and the set forms a commutative ring with unit element with respect to + and this multiplication operation. Now  $Q(\sqrt[3]{3})$  we show that the set Q is an algebra over the field. For this, it is necessary to show that the equality  $\lambda \cdot (x \cdot y) = (\lambda x) \cdot y = x \cdot (\lambda y)$  is fulfilled. But the appropriateness of these conditions is checked directly. Hence,  $Q(\sqrt[3]{3})$  the set  $Q$  forms an algebra over the field. Now we determine its color, that is, the dimension of linear space.

$$\alpha_1 = 1$$

$$\alpha_2 = \sqrt[3]{3}$$

$$\alpha_3 = \sqrt[3]{9}$$

is a system of linear free vectors in element space.  $Q(\sqrt[3]{3})$  If the system formed by adding an arbitrary  $\alpha \in Q(\sqrt[3]{3})$  vector to this system is linearly connected, then the color 3 equality of this algebra is derived. It can be verified that

$$a \alpha_1 + b \alpha_2 + c \alpha_3 + (-1) \alpha = 0$$

equality holds, the system of vectors  $\alpha_1, \alpha_2, \alpha_3, \alpha$  and 4 is linearly connected. So,

$$\dim_{\mathbb{Q}} \mathbb{Q}(\sqrt[3]{3}) = 3$$

equality is appropriate.

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