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ON HOMOMORPHISM AND ISOMORPHISM OF ALGEBRAIC STRUCTURES

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Annotation: The concepts of algebraic structure extension, algebraic structure homomorphism and isomorphism were introduced, rules, ideas were considered.

Key word: Group, ring, field, linear space, algebra, homomorph, isomorph.

If G one or more operations are defined on a set, G the set together with these algebraic operation(s) is called an algebraic structure (system). So, the concepts of group, ring, body, field, linear space, algebra are different representations of algebraic structures.
If $G_1 \subset G$ an algebraic structure forms, then it G_1 is also called a partial structure of , or an extension of the G structure G of . G_1
For example, Z the sets R and R are groups by simple addition, and $Z \subset R$ the relation holds, i.e., Z a group R is a subset of a group, R which in turn Z is called an extension of a group. Similarly, the sets Z and R form a ring with respect to simple addition and multiplication, the ring *Z is a partial ring of the ring R* , or the ring R Z is an extension of the ring.
If the set G is a group with G respect to the * operation , G_1 and $\circ$ if the equality holds for G arbitrary elements *α,β of a group* for a reflection $\varphi(a*b) = \varphi(a) \circ \varphi(b)$ that transfers $\varphi : G \to G_1$ a group to a group G_1, then φ the reflection is called a homomorphic reflection or homomorphism. In this

$$\varphi(G) = \{x \in G_1 \,|\, \exists a \in G, \varphi(a) = x\}$$

the set is called the homomorphic image (image) of the group *G.*
that is mutually one-valued $\varphi : G \to G_1$ is called an isomorphic reflection. In this case, the groups *G* and G_1 are called isomorphic groups.
K and K_1 form a ring $\varphi : K \to K_1$ to reflect

and
$$\left.\begin{aligned} \varphi(a+b) &= \varphi(a) + \varphi(b) \\ \varphi(a \cdot b) &= \varphi(a) \cdot \varphi(b) \end{aligned}\right\} (2)$$

if conditions are met, φ reflection is called homomorphic reflection. (2) + and on the left side of the equation· It should be noted that the actions + and on the right side of ·*K are operations in the* K_1 ring.
Homomorphisms and isomorphisms of linear spaces and algebras are also defined by making appropriate changes in the above definitions.
For example $\varphi : L \to L_1$ –to be a homomorphism between reflection L and linear spaces L_1

$$\left.\begin{aligned} \varphi(x+y) &= \varphi(x) + \varphi(y) \\ \varphi(\lambda x) &= \lambda \varphi(x) \end{aligned}\right\} (3)$$

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must satisfy the conditions. $\varphi:\lambda \to L_1$reflection R is for an isomorphic reflection between given algebras over a field

$$\left.\begin{array}{l}\varphi(x+y)=\varphi(x)+\varphi(y)\\ \varphi(\lambda\cdot x)=\lambda\varphi(x)\\ \varphi(x\cdot y)=\varphi(x)\cdot\varphi(y)\end{array}\right\}\text{(4) conditions must be fulfilled.}$$

Here also + and , on the left-hand sides of equations (3) and (4) $\cdot$ and λ scalar multiplication operations L in the set, + and , on the right $\cdot$ and actions L_1 are actions in

Examples.

1 Z The set of -integers and $M_2(\mathrm{Z})$-the set of all matrices with second-order integer elements form a loop with respect to simple addition and multiplication operations. Is Z the ring $M_2(\mathrm{Z})$ an extension of the ring?

Solution: Z a matrix of the form $M_2(\mathrm{Z})$ formed by $\begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}$any integer from the set κ belongs to the ring. If conditionally $k=\begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}$that

if we accept , $\mathrm{Z}\subset \mathrm{M}_2(\mathrm{Z})$the relation holds and

$$k+n=\begin{pmatrix} k+n & 0 \\ 0 & 0 \end{pmatrix}$$

$$k\cdot n=\begin{pmatrix} k\cdot n & 0 \\ 0 & 0 \end{pmatrix}$$

equalities are fulfilled, that is, k a number and its other form, $\begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}$the matrix, have the same properties. So $\mathrm{M}_2(\mathrm{Z})$the loop Z is an extension of the loop. In addition, $\mathrm{M}_2(\mathrm{Z})$there are elements $\mathrm{Z}\neq \mathrm{M}_2(\mathrm{Z})$that do not belong to $\begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}$da , which are not visible, ie Z

2 The set of real numbers R forms a group with respect to simple addition, and the set of all real numbers different from O forms a group with respect to multiplication. If we define $\varphi: R \to R_\circ$ reflection $\varphi(x)=3^x$by the equality

a) Is φ acceleration homomorphic ?

c) Is φ acceleration isomorphic?

c) Construct a homomorphic representation φ of (R) .

Solving _ a) $\varphi(x+y)=3^{x+y}=3^x\cdot 3^y=\varphi(x)\cdot\varphi(y)$So φ will be a homomorphic reflection.

c) If $x\neq y$ fulfilled $\varphi(x)\neq\varphi(y)$ is also done. But φ the reflection is not mutually exclusive, because of $R_\circ$ the negative elements in the group φ There is no actual number that the mirroring aid will pass. So, φ–isomorphism does not occur.

s) $\varphi: R\to R_\circ$-we construct a homomorphic reflection image. $y(x)=3^3$it follows from equality . $3^x>0$Hence, $\varphi: R\to R_+$-can consist of the set of all positive numbers. To prove the last conclusion, it suffices to show that there is an element of the group which goes by reflection to R any $\alpha\in R_+$positive number φ. To do this, you need to solve $3^x=\alpha$the equation with respect to x .

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Hence, $x = \log_3 \alpha \in R$ the $\varphi(x) = \alpha$ equality holds. Therefore, $\varphi(R) = R_+$ equality is appropriate

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