International Journal on Integrated Education
https://journals.researchparks.org/index.php/IJIE
e-ISSN : 2620 - 3502
p-ISSN : 2615 - 3785

ON THE CANONICAL FORMATION OF CERTAIN SECOND-ORDER PARTIAL DIFFERENTIAL DIFFERENTIAL EQUATIONS IN A TYPE PRESERVING FIELD

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Key words: Function, multivariable function, derivative, eigenvalue, differential equation,

Abstract

Describes the canonical representation of some second-order partial differential equations in a type-preserving domain.

Definition: E^2 let there be $u_{xy} = u_{yx}$ some function with second-order eigenderivatives in the space ($u(x, y)$). In that case

$$F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0 \quad (1)$$

the equation is called a differential equation with a given particular derivative in general.
Here F is some function.
A similar polynomial variable second-order partial differential equation is expressed in the following form:

$$F(x_1, x_2, \ldots, x_n, u, u_{x_1}, u_{x_2}, \ldots, u_{x_n}, \ldots, u_{x_i x_j}, \ldots) = 0. \quad (2)$$

Definition: A second-order partial differential equation is said to be linear with respect to higher-order derivatives if it has the following form with respect to higher-order derivatives:

$$a_{11}(x, y)\cdot u_{xx} + 2a_{12}(x, y)\cdot u_{xy} + a_{22}(x, y)\cdot u_{yy} + F(x, y, u, u_x, u_y) = 0. \quad (3)$$

Description:

$$a_{11}dy^2 - 2a_{12}dxdy + a_{22}dx^2 = 0(4)$$

equation (3) is called the characteristic equation of equation.

Definition: Equation (3) is of the hyperbolic type if it is in D some D sphere , if it is $a_{12}^2 - a_{11}\cdot a_{22} > 0$ in the sphere $a_{12}^2 - a_{11}\cdot a_{22} < 0$, the given equation (3) is of the elliptic type, and if it is D in the sphere $a_{12}^2 - a_{11}\cdot a_{22} = 0$, it is said to be of the parabolic type. Thus , $a_{12}^2 - a_{11}\cdot a_{22}$ depending on the sign of the expression, equation (3) can be reduced to the following canonical forms.

$a_{12}^2 - a_{11}\cdot a_{22} > 0$ (in hyperbolic form), $u_{xx} - u_{yy} = \Phi$ or $u_{xy} = \Phi$.

$a_{12}^2 - a_{11}\cdot a_{22} < 0$ (in elliptical form), $u_{xx} + u_{yy} = \Phi$.

$a_{12}^2 - a_{11}\cdot a_{22} = 0$ (parabolic type) $u_{xx} = \Phi$.

Here F is the function resulting from the simplification.

<u>1- Example.</u> Let's convert the following equation into canonical form:

$u_{xx} - 2u_{xy} - 3u_{yy} + u_y = 0.$

$a_{12} = -1, a_{11} = 1,\ a_{22} = -3$ are the coefficients of the equation. $\Delta = a_{12}^2 - a_{11}\cdot a_{22}$ we calculate the

e-ISSN : 26203502
p-ISSN : 26153785

value of the expression. $\Delta = 4 > 0$, so the equation belongs to the hyperbolic type. (4) we solve the characteristic equation.

$$\frac{dy}{dx} = \frac{-1+2}{1} = 1 \Rightarrow x - y = C, \frac{dy}{dx} = \frac{-1-2}{1} = -3 \Rightarrow 3x + y = C$$

one of the general integrals ξ and the other η by , applying the results of calculations using the formulas to the given equation, after simplifications we create the following canonical representation of the equation:

$$u_{\xi\eta} - \frac{1}{16}(u_\xi - u_\eta) = 0.$$

<u>2- Example.</u> Given the following equation:

$$u_{xx} + 2u_{xy} + 2u_{yy} + 4u_{yz} + 5u_{zz} = 0.$$

The characteristic fit to this equation $Q = \lambda_1^2 + 2\lambda_1\lambda_2 + 2\lambda_2^2 + 4\lambda_2\lambda_3 + 5\lambda_3^2$ has a quadratic form. Let's make this quadratic form canonical using, for example, the Lagrange method: $Q = (\lambda_1 + \lambda_2)^2 + (\lambda_2 + 2\lambda_3)^2 + \lambda_3^2$. We enter the following definitions:

$$\mu_1 = \lambda_1 + \lambda_2;\ \mu_2 = \lambda_2 + 2\lambda_3;\ \mu_1 = \lambda_3 \qquad (5)$$

and as a result we bring the form Q into the canonical form: $Q = \mu_1^2 + \mu_2^2 + \mu_3^2$. (5) λ we can find . Thus, $M = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ the following characteristic affine permutations with matrices: $\lambda_1 = \mu_1 - \mu_2 + 2\mu_3,\ \ \lambda_2 = \mu_2 - 2\mu_3, \lambda_3 = \mu_3$ Q makes the form canonical: $Q = \mu_1^2 + \mu_2^2 + \mu_3^2$.

The matrix of the characteristic affine substitution, which brings the given differential equation into the canonical form, is a matrix symmetric to the matrix $M^* = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -2 & 1 \end{pmatrix} M :$, this substitution has the following form:

$$\xi = x;\ \eta = -x + y;\ \zeta = 2x - 2y + z.$$

these and $u(x, y, z) = v(\xi, \eta)$ notation, we find:

$$u_{xx} = v_{\xi\xi} + v_{\eta\eta} + 4v_{\zeta\zeta} - 2v_{\xi\eta} + 4v_{\xi\zeta} - 4v_{\eta\zeta};$$

$$u_{yy} = v_{\eta\eta} + 4v_{\zeta\zeta} - 4v_{\eta\zeta}; \qquad u_{zz} = v_{\zeta\zeta};$$

$$u_{xy} = -v_{\eta\eta} - 4v_{\zeta\zeta} + v_{\xi\eta} - 2v_{\xi\zeta} + 4v_{\eta\zeta};\ u_{yz} = -2v_{\zeta\zeta} + v_{\eta\zeta}.$$

After putting the found expressions into the equation and performing simplifications, we get the canonical representation of the given equation:

$$v_{\xi\xi} + v_{\eta\eta} + v_{\zeta\zeta} = 0.$$

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e-ISSN : 26203502
p-ISSN : 26153785

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