ON THE CANONICAL FORMATION OF CERTAIN SECOND-ORDER PARTIAL DIFFERENTIAL DIFFERENTIAL EQUATIONS IN A TYPE PRESERVING FIELD

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Abstract
Describes the canonical representation of some second-order partial differential equations in a type-preserving domain.

Definition: $E^2$ let there be $u_{xy} = u_{yx}$ some function with second-order eigenderivatives in the space $(u(x, y))$. In that case

$$F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0$$  \hspace{1cm} (1)

the equation is called a differential equation with a given particular derivative in general. Here $F$ is some function.

A similar polynomial variable second-order partial differential equation is expressed in the following form:

$$F(x_1, x_2, \ldots, x_n, u, u_{x_1}, u_{x_2}, \ldots, u_{x_n}, \ldots, u_{x_1 x_2}, \ldots) = 0. \hspace{1cm} (2)$$

Definition: A second-order partial differential equation is said to be linear with respect to higher-order derivatives if it has the following form with respect to higher-order derivatives:

$$a_{11}(x, y)u_{xx} + 2a_{12}(x, y)u_{xy} + a_{22}(x, y)u_{yy} + F(x, y, u, u_x, u_y) = 0. \hspace{1cm} (3)$$

Description:

$a_{11}d^2y^2 - 2a_{12}dxdy + a_{22}dx^2 = 0(4)$

Equation (3) is called the characteristic equation of equation.

Definition: Equation (3) is of the hyperbolic type if it is in $D$ some $D$ sphere, if it is $a_{12}^2 - a_{11} \cdot a_{22} > 0$ in the sphere $a_{12}^2 - a_{11} \cdot a_{22} < 0$, the given equation (3) is of the elliptic type, and if it is $D$ in the sphere $a_{12}^2 - a_{11} \cdot a_{22} = 0$, it is said to be of the parabolic type. Thus, $a_{12}^2 - a_{11} \cdot a_{22}$ depending on the sign of the expression, equation (3) can be reduced to the following canonical forms.

$$a_{12}^2 - a_{11} \cdot a_{22} > 0 \text{ (in hyperbolic form), } u_{xx} - u_{yy} = \Phi \text{ or } u_{xy} = \Phi.$$  
$$a_{12}^2 - a_{11} \cdot a_{22} < 0 \text{ (in elliptical form), } u_{xx} + u_{yy} = \Phi .$$  
$$a_{12}^2 - a_{11} \cdot a_{22} = 0 \text{ (parabolic type) } u_{xx} = \Phi .$$

Here $F$ is the function resulting from the simplification.

1- Example. Let's convert the following equation into canonical form:

$$u_{xx} - 2u_{xy} - 3u_{yy} + u_y = 0.$$

$a_{12} = -1, a_{11} = 1, a_{22} = -3$ are the coefficients of the equation. $\Delta = a_{12}^2 - a_{11} \cdot a_{22}$ we calculate the
value of the expression. \( \Delta = 4 > 0 \), so the equation belongs to the hyperbolic type. ( 4 ) we solve the characteristic equation.

\[
\frac{dy}{dx} = \frac{-1+2}{1} = 1 \Rightarrow x - y = C, \quad \frac{dy}{dx} = \frac{-1-2}{1} = -3 \Rightarrow 3x + y = C
\]

one of the general integrals \( \xi \) and the other \( \eta \) by , applying the results of calculations using the formulas to the given equation, after simplifications we create the following canonical representation of the equation:

\[
u_{\xi \eta} - \frac{1}{16} (u_{\xi} - u_{\eta}) = 0.
\]

2- Example. Given the following equation:

\[
uxx + 2uxy + 2uyy + 4uyz + 5uzz = 0.
\]

The characteristic fit to this equation \( Q = \lambda_1^2 + 2\lambda_1\lambda_2 + 2\lambda_2^2 + 4\lambda_2\lambda_3 + 5\lambda_3^2 \) has a quadratic form. Let's make this quadratic form canonical using, for example, the Lagrange method:

\[
Q = (\lambda_1 + \lambda_2)^2 + (\lambda_2 + 2\lambda_3)^2 + \lambda_3^2.
\]

We enter the following definitions:

\[
\mu_1 = \lambda_1 + \lambda_2; \quad \mu_2 = \lambda_2 + 2\lambda_3; \quad \mu_3 = \lambda_3
\]

and as a result we bring the form \( Q \) into the canonical form:

\[
Q = \mu_1^2 + \mu_2^2 + \mu_3^2.
\]

( 5 ) \( \lambda \) we can find. Thus, \( M = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \) the following characteristic affine permutations with matrices:

\[
\lambda_1 = \mu_1 - \mu_2 + 2\mu_3, \quad \lambda_2 = \mu_2 - 2\mu_3, \quad \lambda_3 = \mu_3
\]

\( Q \) makes the form canonical:

\[
Q = \mu_1^2 + \mu_2^2 + \mu_3^2.
\]

The matrix of the characteristic affine substitution, which brings the given differential equation into the canonical form, is a matrix symmetric to the matrix \( M^* = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -2 & 1 \end{pmatrix} M : \), this substitution has the following form:

\[
\xi = x; \quad \eta = -x + y; \quad \zeta = 2x - 2y + z.
\]

these and \( u(x,y,z) = v(\xi, \eta) \) notation, we find:

\[
uxx = v_{\xi \xi} + v_{\eta \eta} + 4v_{\zeta \zeta} - 2v_{\xi \eta} + 4v_{\xi \zeta} - 4v_{\eta \zeta};
\]

\[
uxy = v_{\eta \eta} + 4v_{\zeta \zeta} - 4v_{\eta \zeta}; \quad u_{yy} = v_{\zeta \zeta};
\]

\[
u_{xy} = -v_{\eta \eta} - 4v_{\zeta \zeta} + v_{\xi \eta} - 2v_{\xi \zeta} + 4v_{\eta \zeta}; \quad u_{yz} = -2v_{\xi \zeta} + v_{\eta \zeta}.
\]

After putting the found expressions into the equation and performing simplifications, we get the canonical representation of the given equation:

\[
v_{\xi \xi} + v_{\eta \eta} + v_{\zeta \zeta} = 0.
\]

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