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On some methods of solving one-variable inequalities involving modulus notation

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Abstract. The methods of solving equations and inequalities involving the modulus symbol are somewhat fewer and more difficult, and students are more interested in learning the easier methods of solving them. One such method is the interval method.

Keywords. *Module, equation, inequality, method of intervals, domain of values, domain of definition.*

The method of intervals is one of the main methods of solving equations and inequalities or their systems, and this method is not well covered in methodological literature.

This work describes some easier and simpler ways to solve inequalities involving the modulus sign by the method of intervals.

Theorem 1. If optional $x \in M$ and $f(x)$ only $g(x)$ takes non-negative values, M then in the set:

$$f(x) \vee g(x) \Leftrightarrow (f(x))^2 \vee (g(x))^2$$

or

$$|f(x)| \vee |g(x)| \Leftrightarrow (f(x))^2 \vee (g(x))^2 .$$

Theorem 2. 1) $g(x) \leq 0$, then $|f(x)| < g(x)$ the solution of the inequality is an empty set; 2) $g(x) > 0$ if

$$|f(x)| < g(x) \Leftrightarrow -g(x) < f(x) < g(x) \Leftrightarrow \begin{cases} f(x) > -g(x) \\ f(x) < g(x). \end{cases}$$

Theorem 3. 1) $g(x) < 0$ if

$$|f(x)| > g(x)$$

The solution of the inequality $D(T) = D(f) \cap D(g)$ consists of;

2) $g(x) = 0$ if

$$|f(x)| > g(x) \Leftrightarrow |f(x)| > 0 \Leftrightarrow f(x) \neq 0 \Leftrightarrow \begin{cases} f(x) > 0, \\ f(x) < 0; \end{cases}$$

3) $g(x) > 0$ if

$$|f(x)| > g(x) \Leftrightarrow \begin{cases} f(x) > g(x), \\ f(x) < -g(x). \end{cases}$$

Example 1. Solve the inequality: $|x+1| + |x| + |x-2| > 5$.

Solution: $x+1=0, x=0,\ x-2=0$ the roots of the equations are corresponding $-1, 0, 2$, and the

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field of determination of the inequality $(-\infty;-1),[-1;0),[0;2),[2;\infty)$ is reduced to intervals. We present two ways to solve this inequality.

Method 1. (A method of constructing a set of systems of inequalities).

$$\left[\begin{array}{l}\begin{cases}x<-1,\\(-x-1)+(-x)+(-x+2)>5;\end{cases}\\\begin{cases}-1\le x<0,\\(x+1)+(-x)+(-x+2)>5;\end{cases}\\\begin{cases}0\le x<2,\\(x+1)+x+(-x+2)>5;\end{cases}\\\begin{cases}x\ge 2,\\(x+1)+x+(x-2)>5;\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}x<-1,\\-3x+1>5;\end{cases}\\\begin{cases}-1\le x<0,\\-x+3>5;\end{cases}\\\begin{cases}0\le x<2,\\x+3>5;\end{cases}\\\begin{cases}x\ge 2,\\3x-1>5;\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}x<-1,\\x<-1\frac{1}{3};\end{cases}\\\begin{cases}-1\le x<0,\\x<-2;\end{cases}\\\begin{cases}0\le x<2,\\x>2;\end{cases}\\\begin{cases}x\ge 2,\\x>2;\end{cases}\end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{l}x<-1\frac{1}{3},\\\varnothing,\\\varnothing,\\x>2;\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x<-1\frac{1}{3},\\x>2.\end{array}\right. \Rightarrow x\in(-\infty;-1\frac{1}{3})\bigcup(2;\infty).$$

Method 2. (Table method). In the 1st line of the table, the intervals are written, in the following lines the values of the expressions under the absolute value sign in this interval, then the expression of the inequality in each of the resulting intervals is written and solved. Finally, the solutions in the appropriate intervals are determined and the answer is written.

Intervals / Functions	$(-\infty;-1)$	$[-1;0)$	$[0;2)$	$[2;\infty)$
$\lvert x+1\rvert$	$-x-1$	$x+1$	$x+1$	$x+1$
$\lvert x\rvert$	$-x$	$-x$	x	x
$\lvert x-2\rvert$	$-x+2$	$-x+2$	$-x+2$	$x-2$
$\lvert x+1\rvert+\lvert x\rvert+\lvert x-2\rvert>5$	$-3x+1>5\Leftrightarrow$ $\Leftrightarrow x<-1\frac{1}{3}$	$-x+3>5\Leftrightarrow$ $\Leftrightarrow x<-2$	$x+3>5\Leftrightarrow$ $\Leftrightarrow x>2$	$3x-1>5\Leftrightarrow$ $\Leftrightarrow x>2$
A suitable interval solution	$(-\infty;-1\frac{1}{3})$	$\varnothing$	$\varnothing$	$(2;\infty)$

So, $x\in(-\infty;-1\frac{1}{3})\bigcup(2;\infty)$.

Example 2. Solve the inequality: $|x|+|x-2|<3$.

Solving. $x=0,\ x-2=0$ the roots of the equations are corresponding $0,2$, and the field of determination of the inequality $(-\infty;0),[0;2),[2;\infty)$ is reduced to intervals. We present two ways to solve this inequality.

Method 1. (A method of constructing a set of systems of inequalities).

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$$\left[\begin{array}{l}\begin{cases}x<0,\\ -x-x+2<3;\end{cases}\\ \begin{cases}0\le x<2,\\ x-x+2<3;\end{cases}\\ \begin{cases}x\ge 2,\\ x+x-2<3;\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}x<0,\\ -2x<1;\end{cases}\\ \begin{cases}0\le x<2,\\ 2<3;\end{cases}\\ \begin{cases}x\ge 2,\\ 2x<5;\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}x<0,\\ x>-\frac{1}{2};\end{cases}\\ \begin{cases}0\le x<2,\\ 2<3;\end{cases}\\ \begin{cases}x\ge 2,\\ x<\frac{5}{2};\end{cases}\end{array}\right. \Leftrightarrow x\in(-\frac{1}{2};\frac{5}{2})$$

Method 2. (Table method). In the 1st line of the table, the intervals are written, in the following lines the values of the expressions under the absolute value sign in this interval, then the expression of the inequality in each of the resulting intervals is written and solved. Finally, the solutions in the appropriate intervals are determined and the answer is written.

Intervals / Functions	$(-\infty;0)$	$[0;2)$	$[2;\infty)$
$\lvert x\rvert$	$-x$	x	x
$\lvert x-2\rvert$	$-x+2$	$-x+2$	$x-2$
$\lvert x\rvert+\lvert x-2\rvert<3$	$-2x+2<3\Leftrightarrow$ $\Leftrightarrow x>-\frac{1}{2}$	$2<3$	$2x-2<3\Leftrightarrow$ $\Leftrightarrow x<\frac{5}{2}$
A suitable interval solution	$(-\frac{1}{2};0)$	$[0;2)$	$[2;\frac{5}{2})$

So, $x\in(-\frac{1}{2};\frac{5}{2})$.

References

1. A.Sh. Bloch, G.L. Trukhan Neravenstva, Minsk, "Narodnaya osvita", 1972.
2. A.I. Sharova Uravneniya i neravenstva, Kyiv, "Vysshaya shkola", 1981g.
3. B.M. Sakhaev Reshenie neravenstv metodom promejutkov ravnosilnyx zamen, Tashkent. izd. TashGU, 1982g.