International Journal on Integrated Education
https://journals.researchparks.org/index.php/IJIE
e-ISSN : 2620 - 3502
p-ISSN : 2615 - 3785

Trigonometric equations form replace solve about

Gavhar Akhadovna Akhmedova
Kokan DPI Mathematics department associate professor

Tulasheva Surayyoxon Isomiddin qizi
Kokan DPI listener

Annotation
In the process of solving trigonometric equations, when we make some changes of forms or when the left side is replaced by the right side of the formulas given in the article, the solution may disappear. This is why it is important to check the roots that we find when using these formulas.

Key words: equation, change of form, field of determination, root, solution, system of inequalities **.**

Some one trigonometric equations solve in the process roots lost to stay cases happened will give . This left and right parts each different identification to the field have has been formulas apply as a result happen will be For example the following formulas :

$$tg\alpha=\frac{1}{ctg\alpha};\ ctg\alpha=\frac{1}{tg\alpha};\ tg\left(\alpha+\beta\right)=\frac{tg\alpha+tg\beta}{1-tg\alpha tg\beta};\ tg\left(\alpha-\beta\right)=\frac{tg\alpha-tg\beta}{1+tg\alpha tg\beta};$$

$$\sin\alpha=\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}};\ \ \cos\alpha=\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}};\ tg\alpha=\frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}$$

and others _ $tg\alpha=\frac{1}{ctg\alpha}$ the left side of the formula $\alpha\neq\frac{\pi}{2}+\pi k;\ k\in Z$ at, right part $\alpha\neq\frac{\pi}{2}\cdot m;\ m\in Z$ defined in . $ctg\alpha=\frac{1}{tg\alpha}$ on the left side $\alpha\neq\pi n;\ n\in Z$ of the formula , on the right part $\alpha\neq\frac{\pi}{2}\cdot m;\ m\in Z$ meaning at have _ The rest formulas the same for comments conduct can _

Trigonometric the left side of Eq shown of formulas right part with when replaced of the solution lost to stay cases happened to give can _ That's why for this formulas when we use his right part unspecified , left part determined of the unknown values checking to see need will be For example ,

$$\sin\alpha=\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}};\ \ \cos\alpha=\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$$

formulas when we use $\alpha\neq\pi+2\pi n;\ n\in Z.$

e-ISSN : 26203502
p-ISSN : 26153785

Example 1. This the equation solve : $\frac{\sin x+\cos x}{\sin x-\cos x}+2tg2x+\cos 2x=0$.

Solving . $\frac{\sin x+\cos x}{\sin x-\cos x}$ of the fraction photo and the denominator $\cos x$ to being and

$$tg2x=\frac{2tgx}{1-tg^2x};\ \cos 2x=\frac{1-tg^2x}{1+tg^2x}$$

formulas supporting

$$\frac{tgx+1}{tgx-1}+\frac{4tgx}{1-tg^2x}+\frac{1-tg^2x}{1+tg^2x}=0 \qquad (1)$$

the equation harvest we do Given of Eq identification field $x\neq\frac{\pi}{4}+\frac{\pi}{2}n;\ n\in Z.$ Harvest has been of Eq identification field as well $x\neq\frac{\pi}{4}+\frac{\pi}{2}n;\ n\in Z,\ \ x\neq\frac{\pi}{2}+\pi k;\ k\in Z.$ Given equation for

$x=\frac{\pi}{2}+\pi k;\ k\in Z$ solution will be (1) equation we solve .

$$\frac{tgx+1}{tgx-1}+\frac{4tgx}{1-tg^2x}+\frac{1-tg^2x}{1+tg^2x}=0;\ \frac{(tgx+1)^2-4tgx}{tg^2x-1}+\frac{1-tg^2x}{1+tg^2x}=0;$$

$$\frac{(tgx-1)^2}{tg^2x-1}-\frac{tg^2x-1}{tg^2x+1}=0;\ \frac{tgx-1}{tgx+1}-\frac{(tgx-1)(tgx+1)}{tg^2x+1}=0,$$

From this $tgx=1,\ x=\frac{\pi}{4}+\pi m;\ m\in Z.$ This is the solution because it wo n't given equation identification to the field belongs to it's not .

$$\frac{1}{tgx+1}-\frac{tgx+1}{tg^2x+1}=0;\ tg^2x+1-tg^2x-2tgx-1=0;$$

$$2tgx=0;\ tgx=0,\ \ x=\pi l;\ l\in Z.$$

Answer : $\pi l;\ \frac{\pi}{2}+\pi k;\ \{k,l\}\in Z$ or $\frac{\pi}{2}p;\ p\in Z.$

Example 2. This the equation solve : $tg\left(x+\frac{\pi}{6}\right)+ctgx=-\sqrt{3}$.

Solving . Given equation for $x\neq\pi n;\ n\in Z,\ x\neq\frac{\pi}{3}+\pi k;\ k\in Z.$ Given the equation the following equation with we will exchange .

$$\frac{tgx+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}tgx}+\frac{1}{tgx}=-\sqrt{3};\ \frac{\sqrt{3}tgx+1}{\sqrt{3}-tgx}+\frac{1}{tgx}=-\sqrt{3} \qquad (2\ )$$

e-ISSN : 26203502
p-ISSN : 26153785

$\frac{\pi}{2}+\pi m;\ m\in Z$ in appearance numbers given of Eq identification to the field corresponding to , but (2) of Eq identification to the field belongs to it's not . That's why for $\frac{\pi}{2}+\pi m;\ m\in Z$ numbers given of Eq root to be that it won't be checking we will see .

$$tg\left(\frac{\pi}{2}+\frac{\pi}{6}+\pi m\right)+ctg\left(\frac{\pi}{2}+\pi m\right)=-\sqrt{3};\quad -ctg\frac{\pi}{6}=-\sqrt{3},\ m\in Z.$$

Example 3. This the equation solve : $\sqrt{-\cos 2x}=\sqrt{ctgx}+1.$

Solving . The equation acceptance to do possible has been values the following inequalities system satisfies .

$$\begin{cases}\cos 2x\le 0,\\ ctgx\ge 0,\end{cases}$$

from this $x\in\left[\frac{\pi}{4}+\pi n,\ \frac{\pi}{2}+\pi n\right];\ n\in Z.$ $\cos 2x=\frac{1-tg^2x}{1+tg^2x}$ and $ctgx=\frac{1}{tgx}$ formulas supporting the following the equation harvest we do :

$$\sqrt{\frac{tg^2x-1}{tg^2x+1}}=\sqrt{\frac{1}{tgx}}+1 \qquad (3)$$

$\frac{1}{\sqrt{tgx}}=y,\ y>0$ designation we enter In that case $tgx=\frac{1}{y^2}$ into equation (3). the following appearance takes :

$\sqrt{\frac{1-y^4}{1+y^4}}=y+1$ or $\frac{1-y^4}{1+y^4}=(y+1)^2$. $y>0$ that it was for

$$(1-y)(1+y^2)=(1+y^4)(y+1);\ 1+y^2-y-y^3=y+1+y^5+y^4;$$

$y^5+y^4+y^3-y^2+2y=0.$ $y>0$ that it was for

$$y^4+y^3+y^2-y+2=0.$$

This is an equation positive to the roots have not because _ $y>0$ at $y^4+y^3>0$ and $y^2-y+2>0.$ Eq in solving $x\ne\frac{\pi}{2}+\pi n;\ n\in Z$ has been from formulas was used . Check as a result $\frac{\pi}{2}+\pi n;\ n\in Z$ in appearance numbers of Eq root will be

Answer : $\frac{\pi}{2}+\pi n;\ n\in Z.$

Example 4. This the equation solve : $\frac{\sin x}{\cos^3 x+\sin^3 x}+tgx=2$.

Solving . $x\ne\frac{\pi}{2}+\pi n;\ n\in Z,\ x\ne-\frac{\pi}{4}+\pi m;\ m\in Z$. Eq form we will exchange .

e-ISSN : 26203502
p-ISSN : 26153785

$$\frac{\frac{\sin x}{\sin^3 x}}{\frac{\cos^3 x+\sin^3 x}{\sin^3 x}}+\frac{1}{ctgx}=2;\ \frac{1+ctg^2x}{1+ctg^3x}+\frac{1}{ctgx}=2$$

The last one in Eq $x\neq\pi k;\ k\in Z.$ Inspections $x=\pi k;\ k\in Z$ numbers given the equation not satisfied let's see can _ Harvest has been the equation we solve .

$$ctgx+ctg^3x+1+ctg^3x=2ctgx+2ctg^4x;\ 2ctg^4x-2ctg^3x+ctgx-1=0;$$
$$\left(2ctg^3x+1\right)\left(ctgx-1\right)=0.$$

From this $ctgx=-\frac{1}{\sqrt[3]{2}}$ or $ctgx=1.$

to Eq we will come As a result $x=-arcctg\frac{1}{\sqrt[3]{2}}+\pi p,\ x=\frac{\pi}{4}+\pi q;\ \{p,q\}\in Z.$

Answer : $-arcctg\frac{1}{\sqrt[3]{2}}+\pi p,\ \frac{\pi}{4}+\pi q;\ \{p,q\}\in Z.$

Example 5. This the equation solve : $15\sin^2 x+2tgx=ctg\left(\frac{\pi}{4}+x\right)-1.$

Solving . Eq identification field we find :

$$x\neq\frac{\pi}{2}+\pi n;\ n\in Z,\ x\neq-\frac{\pi}{4}+\pi k;\ k\in Z.$$
$$\sin^2 x=\frac{1}{1+ctg^2x};\ \ tgx=\frac{1}{ctgx};\ \ ctg\left(\frac{\pi}{4}+x\right)=\frac{ctgx-1}{ctgx+1}$$

formulas attention take

$$\frac{15}{1+ctg^2x}+\frac{2}{ctgx}+\frac{2}{ctgx+1}=0$$

the equation harvest we do In this above identification to the field again will $x\neq\pi m;\ m\in Z$ also be added . $x=\pi m;\ m\in Z$ given equation root to be or that it won't be checking we will see .

$$15\sin^2\pi m+2tg\pi m=ctg\left(\frac{\pi}{4}+\pi m\right)-1,\ m\in Z$$

at equality right , then $x=\pi m;\ m\in Z$ given equation the solution will be $ctgx=y$ replacement we will do it .

$$\frac{15}{1+y^2}+\frac{2}{y}+\frac{2}{y+1}=0.$$

From this $15y^2+15y+2\left(y^3+y^2+y+1\right)+2y\left(1+y^2\right)=0;$

$$4y^3+17y^2+19y+2=0;\ 4y^3+32+17y^2-68+19y+38=0;$$
$$4(y+2)\left(y^2-2y+4\right)+17(y-2)(y+2)+19(y+2)=0;\ (y+2)\left(4y^2+9y+1\right)=0;$$

e-ISSN : 26203502
p-ISSN : 26153785

$y+2=0$ or $4y^2+9y+1=0;\ y=-2$ or $y=\frac{-9\pm\sqrt{65}}{8}$.

In that case $x=arcctg\left(-2\right)+\pi p;\ p\in Z,\ \ x=arcctg\,\frac{-9\pm\sqrt{65}}{8}+\pi q;\ q\in Z.$

Answer : $\pi m;\ m\in Z,\ arcctg\left(-2\right)+\pi p;\ p\in Z,\ arcctg\,\frac{-9\pm\sqrt{65}}{8}+\pi q;\ q\in Z.$

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