INTERNATIONAL JOURNAL ON ORANGE TECHNOLOGIES
www.journalsresearchparks.org/index.php/IJOT e-ISSN: 2615-8140|p-ISSN: 2615-7071
Volume: 03 Issue: 05 | May 2021

Extreme issues related to irrational functions and geometric methods for solving equations

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ABSTRACT

We all know that there are several algebraic ways to solve irrational equations and extreme problems. Solving examples and problems in geometric ways helps to increase students' interest in mathematics and to see more clearly the relationship between geometry and algebra. In this article, a triangle area is used to solve an irrational equation, the cosine theorem, and vectors are used to solve extreme problems.

Keywords: cosine theorem, triangle area, a vector length, semicircle, minimum and maximum values.

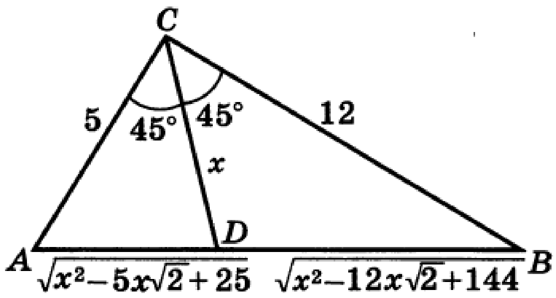
Introduction

There are such extreme problems with irrational equations and irrational expressions that it takes a long time to solve them algebraically. It is much easier to solve them geometrically. Solving examples and problems geometrically helps to increase students' interest in mathematics and to give them a clearer idea of the relationship between geometry and algebra. In this article, the formula for finding the triangle area for solving an irrational equation, the cosine theorem for solving extreme problems, and vectors are used.

Example 1. Solve the equation $\sqrt{x^2-5x\sqrt{2}+25}+\sqrt{x^2-12x\sqrt{2}+144}=13$.

Solution. To solve this equation in the traditional way, each side is squared until the roots disappear. Solving the equation geometrically is done in a short time. In figure 1 $AB=13$ according to Pythagorean theorem from ΔABC.

$D \in AB$, because $\min f(x)=\min(AD+DB)=AB$.



1-figure

Here $f(x)=\sqrt{x^2-5x\sqrt{2}+25}+\sqrt{x^2-12x\sqrt{2}+144}$. If the equation has roots, then they are positive. (The left-hand side value of $x \le 0$ equation is not less than 12).

$$S_{\Delta ABC}=\frac{1}{2}\cdot 5\cdot 12=30.$$

$$S_{\Delta ACD}=\frac{1}{2}\cdot 5\cdot x\cdot \sin 45^0=\frac{5x\sqrt{2}}{4}.$$

$$S_{\Delta BCD}=\frac{1}{2}\cdot 12\cdot x\cdot \sin 45^0=3x\sqrt{2}.$$

So, $30=\frac{5x\sqrt{2}}{4}+3x\sqrt{2}$; $\quad x=\frac{60\sqrt{2}}{17}$.

Answer: $\frac{60\sqrt{2}}{17}$.

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Example 2. a For each value of parameter a, $\sqrt{a^2 - tg^2 z} = 1 - tgz$. In the equation, tgz takes several different values.

Solution. This is not a simple equation. It's not easy to solve in the traditional way, If we replace tgz with x, it will be easier to solve it geometrically. Figure 2 shows graphs of the functions $y = \sqrt{a^2 - x^2}$ (a semicircle in the upper hemisphere with radius $|a|$ which center is at the beginning of coordinates) and $y = 1 - x$ $(x \le 1)$ (light passing through point (0,1) at point (1; 0).

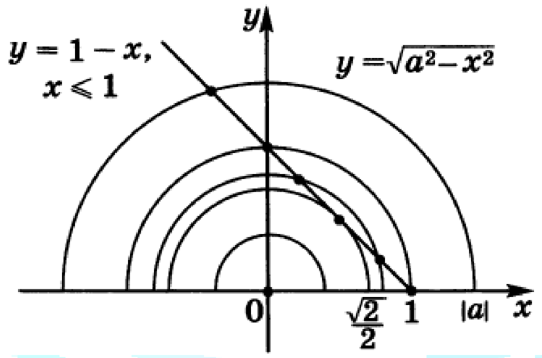


figure 2

So, $\sqrt{a^2 - x^2} = 1 - x$

has no root at $|a| < \frac{\sqrt{2}}{2}$.

has 1 root at $|a| = \frac{\sqrt{2}}{2}$ and $|a| > 1$.

has 2 roots at $\frac{\sqrt{2}}{2} < |a| \le 1$

So, tgz:

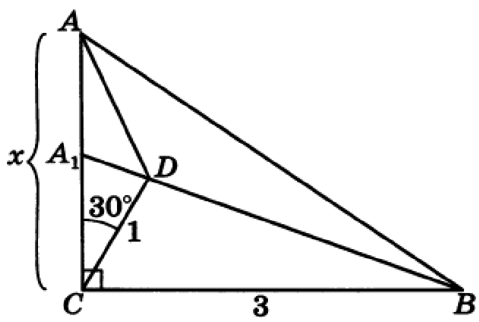
1) It takes a single value at $|a| = \frac{\sqrt{2}}{2}$. (If $-\frac{\sqrt{2}}{2}$ is one number, then $\frac{\sqrt{2}}{2}$ is another number)

2) It takes a single value at $|a| > 1$.

3) it takes 2 different values at $\frac{\sqrt{2}}{2} < |a| \le 1$.

Example3. Find the largest value of the $f(x) = \sqrt{x^2 + 9} - \sqrt{x^2 - x\sqrt{3} + 1}$ function.

Solution. Let's look at the ABC triangle. Here $\angle ACB = 90^0$, $\angle ACD = 30^0$, $AC = x$, $BC = 3$, $CD = 1$ and D point lies inside the triangle ABC. (3-figure)



3-figure

From the triangle ABC by the Pythagorean theorem $AB = \sqrt{x^2 + 9}$. According to the cosine theorem from ΔACD is $AD = \sqrt{x^2 - x\sqrt{3} + 1}$.

$$\max f(x) = \max(AB - AD) = A_1B - A_1D = DB,$$

here $A_1 \in AC$ (if $D \in AB$).

According to the cosine theorem from ΔBCD

$$DB = \sqrt{1^2 + 3^2 - 2 \cdot 1 \cdot 3\cos 60^0} = \sqrt{7}.$$

Answer: $\sqrt{7}$.

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Example 4. Find the smallest value of the function $f(x)=x\sqrt{2}+\sqrt{2x^2+2x+1}+\sqrt{2x^2-14x+25}+\sqrt{2x^2-26x+89}$

Solution. We look at the vectors $\vec{a}(x;x)$, $\vec{b}(4-x;3-x)$, $\vec{c}(x+1;x)$, $\vec{d}(5-x;8-x)$ and their modulus.

$$|\vec{a}|=x\sqrt{2}.$$

$$|\vec{b}|=\sqrt{2x^2-14x+25}.$$

$$|\vec{c}|=\sqrt{2x^2+2x+1}.$$

$$|\vec{d}|=\sqrt{2x^2-26x+89}.$$

$$f(x)=|\vec{a}|+|\vec{b}|+|\vec{c}|+|\vec{d}|.$$

$$|\vec{a}|+|\vec{b}|+|\vec{c}|+|\vec{d}|\geq|\vec{a}+\vec{b}+\vec{c}+\vec{d}|$$ because

$$\min f(x)=|\vec{a}+\vec{b}+\vec{c}+\vec{d}|.$$

$$\vec{a}+\vec{b}+\vec{c}+\vec{d}=\overrightarrow{(10;11)}$$

$$|\vec{a}+\vec{b}+\vec{c}+\vec{d}|=\sqrt{221}.$$

So, $\min f(x)=\sqrt{221}$.

Answer: $\sqrt{221}$.

Example 5. Find the smallest value of the $f(x)=\sqrt{2x^2-4x+2}+\sqrt{2x^2}+\sqrt{2x^2-2x+2}$ function.

Solution. We consider 4 distances O(0;0), A(1;1), $B\left(\frac{1-\sqrt{3}}{2};\frac{1+\sqrt{3}}{2}\right)$, $M(x;x)$ and OM, AM and BM.

$OM=\sqrt{2x^2}$, $AM=\sqrt{2x^2-4x+2}$, $BM=\sqrt{2x^2-2x+2}$.

$$f(x)=OM+AM+BM.$$

It is not difficult to calculate the sides of an triangle ABO.

$AO=BO=AB=\sqrt{2}$. So, ΔABO is equilateral

$$\min(AM+OM+BM)=3R,$$

The radius of the circle drawn outside the R equilateral triangle.

Because $\sqrt{2}=R\sqrt{3}$ it is $\min f(x)=\sqrt{6}$.

Answer: $\sqrt{6}$.

Example 6. If $f(x;y;z)=\sqrt{x^2+1}+\sqrt{y^2+4}+\sqrt{z^2+9}$ and $x+y+z=8$, find the smallest value of the $f(x;y;z)$ function.

Solution.

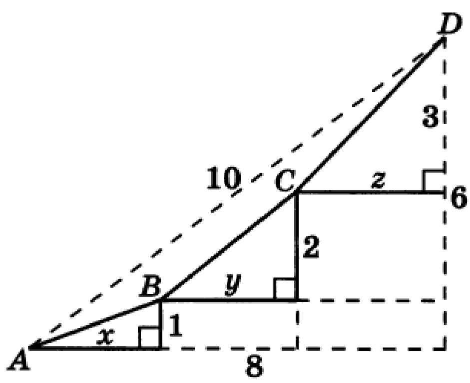

figure 4

According to Figure 4, the length of the $ABCD$ curve is not less than 10, then $f(x;y;z)=10$ at $x+y+z-8$.

Answer: 10.

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