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Investigation on Self-Similar Analysis of the Problem Biological Population Kolmogorov-Fisher Type System

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Abstract: In this work we considered a parabolic system of two quasilinear reaction-diffusion equations for a biological population problem of the Kolmogorov-Fisher type describes the process of a biological population in a nonlinear two-component medium. We studied the qualitative properties of the solution to Cauchy problem based on self-similar analysis and its numerical solutions using the methods of modern computer technologies, to study the methods of linearization to the convergence of the iterative process with further visualization.

Keywords: Cauchy problem, quasilinear, reaction-diffusion, biological population, numerical solutions.

We can consider a parabolic system of two quasilinear reaction-diffusion equations for a biological population problem of the Kolmogorov-Fisher type in the following domain

$Q=\{(t,x):\ 0< t < \infty,\ x\in R^2\}$

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left(D_1 u_1^{\sigma_1}\dfrac{\partial u_1}{\partial x}\right) + k_1(t)u_1\cdot\left(1-u_2^{\beta_1}\right) \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left(D_2 u_2^{\sigma_2}\dfrac{\partial u_2}{\partial x}\right) - k_2(t)u_2\cdot\left(1-u_1^{\beta_2}\right) \end{cases} \quad (1)$$

$$u_1\big|_{t=0} = u_{10}(x),\quad u_2\big|_{t=0} = u_{20}(x), \quad (2)$$

It describes the process of a biological population in a nonlinear two-component medium, the diffusion coefficient of which is equal to $D_1 u_1^{\sigma_1}$ and $D_2 u_2^{\sigma_2}$, $\sigma_1, \sigma_2, \beta_1, \beta_2$ - positive real numbers, $u_1 = u_1(t,x)\geq 0$, $u_2 = u_2(t,x)\geq 0$ - sought solutions.

The Cauchy problem and boundary value problems for system (1) in the one-dimensional and multidimensional cases have been studied by many authors.[1]

The purpose of this work is to study the qualitative properties of the solution to problem (1), (2) based on self-similar analysis and its numerical solutions using the methods of modern computer technologies. Also the article has a purpose to study the methods of linearization to the convergence of the iterative process with further visualization. The main estimations of the solutions and the resulting free boundary have been found, which makes it possible to choose appropriate initial approximations [...] Each of them has their own counting systems.

Now we will start constructing a self-similar system of equations for (1) - (2). It is a simpler system of equations for research.

We construct a self-similar system of equations by the method of nonlinear splitting.

Instead of (1)

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$u_1(t,x) = e^{k_1 t} v_1(t,x),$

$u_2(t,x) = e^{k_2 t} v_2(t,x)$

This will lead (1) to the form:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial x}\left(D_1 v_1^{\sigma_1} \dfrac{\partial v_1}{\partial x} \right) + k_1 e^{((\beta_1+1)k_1-(\delta_1+1)k_2)t} v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial x}\left(D_2 v_2^{\sigma_2} \dfrac{\partial v_2}{\partial x} \right) + k_2(t) e^{((\beta_2+1)k_2-(\delta_2+1)k_1)t} v_1^{\beta_2} v_2, \end{cases} \quad (3)$$

$v_1|_{t=0} = v_{10}(x)$, $v_2|_{t=0} = v_{20}(x)$,

Where $a_1 = (\delta_1 k_1)^{b_1}$, $a_2 = (\delta_2 k_2)^{b_2}$

$b_1 = [(\beta_1+1)k_1 - (\sigma_1+1)]k_2 / \sigma_1 k_1, \quad b_2 = (\beta_2+1)k_2 - (\sigma_2+1)k_1 / \sigma_2 k_2$

After choosing $\sigma_1 k_1 = \sigma_2 k_2$, we can achieve the following forms of equation systems.

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial x}\left(D_1 v_1^{\sigma_1} \dfrac{\partial v_1}{\partial x} \right) - a_1 \tau^{b_1} v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial x}\left(D_2 v_2^{\sigma_2} \dfrac{\partial v_2}{\partial x} \right) - a_2 \tau^{b_2} v_1^{\beta_2} v_2, \end{cases} \quad (4)$$

In the following, we can write one of the ways of auto-model systems for equation systems (4). It is done like in the following:

$$\begin{cases} \dfrac{d\bar{v}_1}{d\tau} = -a_1 \tau^{b_1} v_1 v_2^{\beta_1}, \\ \dfrac{d\bar{v}_2}{d\tau} = -a_2 \tau^{b_2} v_1^{\beta_2} v_2, \end{cases}$$

It has a solution in the following:

$\bar{v}_1(t) = c_1(\tau + T)^{\gamma_1}, \bar{v}_2(t) = c_2(\tau + T)^{\gamma_2}, \ T > 0$,

And then systems of solution are sought in the following steps. (3)-(4)

$v_1(t,x) = \bar{v}_1(t) w_1(\tau, x)$,

$v_2(t,x) = \bar{v}_2(t) w_2(\tau, x)$,

here $\tau = \tau(t)$ is chosen in the following way:

$$\tau(t) = \int \bar{v}_1^{\sigma_1}(t)dt = \frac{1}{\gamma_1\sigma_1 + 1}(T+t)^{\sigma_1\gamma_1+1}, \qquad \gamma_1\sigma_1 + \ \neq$$

$$u\,\tau(t) = \ln(T+t), \qquad \gamma_1\sigma_1 + \ =$$

Here we can choose equation system for $w_i(\tau, x), \ i = 1,2$

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$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial x}(D_1 w_1^{\sigma_1} \dfrac{\partial w_1}{\partial x}) - \theta_1 (w_1 w_2^{\beta_1} - w_1) \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial x}(D_2 w_2^{\sigma_2} \dfrac{\partial w_2}{\partial x}) + \theta_2 (w_2 w_1^{\beta_1} - w_2) \end{cases},$$

$$\eta_1 = b_1 + 1 + \beta_1 (b_2 + 1) \neq 0$$

$$\eta_2 = -\beta_2 (b_1 + 1) + (b_2 + 1) \neq 0$$

It is as $\gamma_1 \sigma_1 > 0$, $\gamma_1 \sigma_1 = \gamma_2 \sigma_2$, $d_i > 0$. In this case we can rely on e $w_i(\tau(t), x) = y_i(\xi), \quad \xi = |x| / \tau_1^{1/2}$, $i = 1, 2$
,

We can choose the following equation system considering the fact that equation for $w_i(\tau, x)$ without little members is always auto-model.

$$\begin{cases} \xi^{1-N} \dfrac{d}{d\xi}(\xi^{N-1} y_1^{\sigma_1} \dfrac{dy_1}{d\xi}) + \dfrac{\xi}{2\theta_1} \dfrac{dy_1}{d\xi} - \mu_1 (y_1 - y_1\, y_2^{\beta_1}) = 0 \\ \xi^{1-N} \dfrac{d}{d\xi}(\xi^{N-1} y_2^{\sigma_2} \dfrac{dy_2}{d\xi}) + \dfrac{\xi}{2\theta_2} \dfrac{dy_2}{d\xi} + \mu_2 (y_2 - y_2\, y_1^{\beta_2}) = 0 \end{cases} \qquad (6.105)$$

Here $\mu_i = \dfrac{1}{\theta_i \sigma_i} \quad \theta_i = \begin{cases} 1, & i = \\ \gamma_1^{-\sigma_1} \gamma_2^{\sigma_2}, & i = \end{cases}$

The study of the qualitative properties of the system (1) - (2) made it possible to perform a numerical experiment depending on the values included in the system of numerical parameters. For this purpose, the constructed asymptotic solutions were used as an initial approximation. In the numerical solution of the problem for the linearization of system (1) - (2), linearizations by the Newton and Picard methods were used. The method of nonlinear splitting is proposed to solve the problem of a biological population.

$$\omega_h = \{x_i = ih,\ h > 0,\ i = 0,1,\ldots,n,\ \ hn = l\},$$

Temporary grid $\omega_{h_1} = \{t_j = jh_1,\ h_1 > 0,\ j = 0,1,\ldots,n,\ \tau m = T\}$.

The main problem in nonlinear problems is the appropriate choice of the initial approximation and the way to linearize equation (3).

We replace problem (3) - (4) with an implicit difference scheme and obtain a difference problem with an error $O(h^2 + h_1)$.

$$\psi_1(t) = \bar{v}_1(t), \quad v_{10}(t,x) = \psi_1(t) \cdot \left(a - \frac{\sigma_1}{4}\xi^2\right)_+^{1/\sigma_1},$$

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$$\psi_2(t) = \bar{v}_2(t), \quad v_{20}(t,x) = \psi_2(t) \cdot \left(a - \frac{\sigma_2}{4} \xi^2 \right)_+^{1/\sigma_2},$$

$$\xi = \frac{x}{[\tau(t)]^{1/2}}, \quad \tau(t) = \int_0^t [\psi(y)]^{\sigma} dy.$$

$(a)_+$ means $(a)_+ = \max(0, a)$.

As a conclusion, he results of numerical experiments have shown the effectiveness of the proposed approach. Asymptotes of various solutions of the system of type (1) - (2) made it possible to simulate the processes of mutual reaction-diffusion in the form of visualization with animation.

All in all, we can emphasize the importance of a joint study of migration and demographic processes. To analyze the population dynamics of interacting populations, it is important to jointly study the processes of fertility, mortality, trophic interactions, and various migrations. The introduction of nonlinearity into migration flows is the first step towards an adequate description of spatio-temporal population dynamics.

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