

Digital Solution and Programming of Two-Dimensional Thermoelastic Dependent Problems for Transversal Isotropic Bodies

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Abstract: The need to solve the practical problems of certain phenomena and processes occurring in nature is one of the most important and topical issues today. Such processes can be expressed in mathematical form, solved in the form of algebraic, integral or differential equations or systems of equations. Currently, the use of software is widely used in solving major scientific and economic problems. When studying the software development of complex processes and tasks, the construction and analysis of mathematical models of these objects is becoming more common.

Keywords: Composition, construction, thermoelastic, thermal conductivity, deformation, mathematical model, dynamic, tensor, square plate.

INTRODUCTION

The use of composite materials in many industries of the country is becoming a modern requirement. Mathematical modeling of thermoelastic states of structures and their elements and determination of numerical solutions are current problems. In mathematical modeling of composite materials, the material is replaced by homogeneous and anisotropic material. It should be noted that temperature and its derivatives are involved in the equation of motion, and deformation is unknown in the equation of thermal conductivity. , is of great benefit in rocketry, machinery, automotive, construction, medicine, and many other fields of manufacturing.

RESEARCH MATERIALS AND METHODOLOGY

The following is a mathematical model of the dynamic relationship of thermoelastic problems for transversal isotropic bodies and the numerical solution of this model. The two-dimensional motion equations of the related dynamic problem for transverse isotropic bodies are as follows:

$$C_{1111} \frac{\partial^2 u}{\partial x^2} + (C_{1122} + C_{1212}) \frac{\partial^2 v}{\partial x \partial y} + C_{1212} \frac{\partial^2 u}{\partial y^2} - \beta_{11} \frac{\partial T}{\partial x} + X_1 = \rho \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$C_{1212} \frac{\partial^2 v}{\partial x^2} + (C_{1212} + C_{2211}) \frac{\partial^2 u}{\partial x \partial y} + C_{2222} \frac{\partial^2 v}{\partial y^2} - \beta_{22} \frac{\partial T}{\partial y} + X_2 = \rho \frac{\partial^2 v}{\partial t^2} \quad (2)$$

Heat dissipation equation for transversal isotropic bodies:

$$\lambda_{11} \frac{\partial^2 T}{\partial x^2} + \lambda_{22} \frac{\partial^2 T}{\partial y^2} - c_\varepsilon \frac{\partial T}{\partial t} - T \left(\beta_{11} \frac{\partial^2 u}{\partial x \partial t} + \beta_{22} \frac{\partial^2 v}{\partial y \partial t} \right) = 0 \quad (3)$$

(3) The initial conditions for this equation are as follows

$$u(x, y, t)|_{t=0} = \varphi_1, \frac{\partial u}{\partial t}|_{t=0} = \psi_1, v(x, y, t)|_{t=0} = \varphi_2, \frac{\partial v}{\partial t}|_{t=0} = \psi_2, T(x, y, t)|_{t=0} = T_0 \quad (4)$$

and the boundary conditions are as follows

$$\begin{aligned} u(x, y, t)|_{x=0} = u_0; \quad u(x, y, t)|_{x=\ell_1} = \bar{u}_0; \quad u(x, y, t)|_{y=0} = u'_0; \quad u(x, y, t)|_{y=\ell_2} = \bar{u}'_0 \quad v(x, y, t)|_{x=0} = v_0; \\ v(x, y, t)|_{x=\ell_1} = \bar{v}_0; \quad v(x, y, t)|_{y=0} = v'_0; \quad v(x, y, t)|_{y=\ell_2} = \bar{v}'_0 \end{aligned} \quad (5)$$

$$T(x, y, t)|_{x=0} = T_1(t); \quad T(x, y, t)|_{x=\ell_1} = T_2(t); \quad T(x, y, t)|_{y=0} = T'_1(t); \quad T(x, y, t)|_{y=\ell_2} = T'_2(t)$$

Here: σ_{ij} - force tensor, X_i - volumetric forces, C_{ijkl} - parameters characterizing the body, ε_{ij} - strain tensor, β_{ij} - coefficient of volumetric thermal expansion, δ_{ij} - Kroneker symbol, here; $\delta_{ij} = \begin{cases} 1 & , i = j \\ 0 & , i \neq j \end{cases}$ c_ε - heat capacity at constant temperature, β_{ij} - thermal expansion tensor, λ_{ij} - heat sink tensor and Koshi relationship, θ - temperature, ρ - density, $t \geq 0, 0 \leq x \leq l_1, 0 \leq y \leq l_2, x = ih_1, (i = \overline{0, k}), y = jh_2 (j = \overline{0, k}), t = n\tau (n = \overline{0, 1, 2, \dots})$ build a family of 3 parallel straight lines at (1) -(3) we replace the equations with their derivatives in different relations.

$$\begin{aligned} C_{1111} \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{h_1^2} + (C_{1122} + C_{1212}) \frac{v_{i+1,j+1}^n - v_{i-1,j+1}^n - v_{i+1,j-1}^n + v_{i-1,j-1}^n}{4h_1h_2} + \\ + C_{1212} \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{h_2^2} - \beta_{11} \frac{T_{i+1,j}^n - T_{i-1,j}^n}{2h_1} = \rho \frac{u_{i,j}^{n+1} - 2u_{i,j}^n + u_{i,j}^{n-1}}{\tau^2} \end{aligned} \quad (6)$$

$$\begin{aligned} C_{2222} \frac{v_{i,j+1}^n + 2v_{i,j}^n + v_{i,j-1}^n}{h_2^2} + (C_{1212} + C_{2211}) \frac{u_{i+1,j+1}^n - u_{i-1,j+1}^n - u_{i+1,j-1}^n + u_{i-1,j-1}^n}{4h_1h_2} + \\ + C_{1212} \frac{v_{i+1,j}^n - 2v_{i,j}^n + v_{i-1,j}^n}{h_1^2} - \beta_{22} \frac{T_{i,j-1}^n - T_{i,j+1}^n}{2h_2} = \rho \frac{v_{i,j}^{n+1} - 2v_{i,j}^n + v_{i,j}^{n-1}}{\tau^2} \end{aligned} \quad (7)$$

$$\begin{aligned} \lambda_{11} \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{h_1^2} + \lambda_{22} \frac{T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n}{h_2^2} - c_\varepsilon \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\tau} - \\ - T_0 (\beta_{11} \frac{u_{i+1,j}^{n+1} - u_{i-1,j}^{n+1} - u_{i+1,j}^{n-1} + u_{i-1,j}^{n-1}}{4h_1\tau} + \beta_{22} \frac{v_{i,j+1}^{n+1} - v_{i,j-1}^{n+1} - v_{i,j+1}^{n-1} + v_{i,j-1}^{n-1}}{4h_2\tau}) = 0 \end{aligned} \quad (8)$$

From the above equations (6) - (7) and (8) - we find $u_{i,j}^{n+1}, v_{i,j}^{n+1}, T_{i,j}^{n+1}$.

$$u_{i,j}^{n+1} = \frac{\tau^2}{\rho} (C_{1111} \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{h_1^2} + (C_{1122} + C_{1212}) \frac{v_{i+1,j+1}^n - v_{i-1,j+1}^n - v_{i+1,j-1}^n + v_{i-1,j-1}^n}{4h_1h_2} + C_{1212} \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{h_2^2} - \beta_{11} \frac{T_{i+1,j}^n - T_{i-1,j}^n}{2h_1}) + 2u_{i,j}^n - u_{i,j}^{n-1}$$
(9)

$$v_{i,j}^{n+1} = \frac{\tau^2}{\rho} (C_{2222} \frac{v_{i,j+1}^n + 2v_{i,j}^n + v_{i,j-1}^n}{h_2^2} + (C_{1212} + C_{2211}) \frac{u_{i+1,j+1}^n - u_{i-1,j+1}^n - u_{i+1,j-1}^n + u_{i-1,j-1}^n}{4h_1h_2} + C_{1212} \frac{v_{i+1,j}^n - 2v_{i,j}^n + v_{i-1,j}^n}{h_1^2} - \beta_{22} \frac{T_{i,j-1}^n - T_{i,j+1}^n}{2h_2}) + 2v_{i,j}^n - v_{i,j}^{n+1}$$
(10)

$$T_{i,j}^{n+1} = \frac{\tau}{c_\epsilon} (\lambda_{11} \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{h_1^2} + \lambda_{22} \frac{T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n}{h_2^2} - T_0 (\beta_{11} \frac{u_{i+1,j}^{n+1} - u_{i-1,j}^{n+1} - u_{i+1,j}^{n-1} + u_{i-1,j}^{n-1}}{4h_1\tau} + \beta_{22} \frac{v_{i,j+1}^{n+1} - v_{i,j-1}^{n+1} - v_{i,j+1}^{n-1} + v_{i,j-1}^{n-1}}{4h_2\tau})) + T_{i,j}^n$$
(11)

Equations (9) - (11) t^{n+1} allow us to find the values of the $u(x, y, t), v(x, y, t), T(x, y, t)$ functions in the layer, if the value of the previous 2 layers is known, ($n=0 \hat{a} \hat{a} n=1$) we find the values of the initial conditions $u(x, y, t)$ and $v(x, y, t)$ functions in the 2 initial layers, and the value of the $T(x, y, t)$ function in layer 1 (11).

$$u_{i,j}^1 = \frac{\tau^2}{\rho} (C_{1111} \frac{u_{i+1,j}^0 - 2u_{i,j}^0 + u_{i-1,j}^0}{h_1^2} + (C_{1122} + C_{1212}) \frac{v_{i+1,j+1}^0 - v_{i-1,j+1}^0 - v_{i+1,j-1}^0 + v_{i-1,j-1}^0}{4h_1h_2} + C_{1212} \frac{u_{i,j+1}^0 - 2u_{i,j}^0 + u_{i,j-1}^0}{h_2^2} - \beta_{11} \frac{T_{i+1,j}^0 - T_{i-1,j}^0}{2h_1}) + 2u_{i,j}^0 - u_{i,j}^{-1}$$
(12)

$$v_{i,j}^1 = \frac{\tau^2}{\rho} (C_{2222} \frac{v_{i,j+1}^0 + 2v_{i,j}^0 + v_{i,j-1}^0}{h_2^2} + (C_{1212} + C_{2211}) \frac{u_{i+1,j+1}^0 - u_{i-1,j+1}^0 - u_{i+1,j-1}^0 + u_{i-1,j-1}^0}{4h_1h_2} + C_{1212} \frac{v_{i+1,j}^0 - 2v_{i,j}^0 + v_{i-1,j}^0}{h_1^2} - \beta_{22} \frac{T_{i,j-1}^0 - T_{i,j+1}^0}{2h_2}) + 2v_{i,j}^0 - v_{i,j}^{-1}$$
(13)

$$T_{i,j}^1 = \frac{\tau}{c_\epsilon} (\lambda_{11} \frac{T_{i+1,j}^0 - 2T_{i,j}^0 + T_{i-1,j}^0}{h_1^2} + \lambda_{22} \frac{T_{i,j+1}^0 - 2T_{i,j}^0 + T_{i,j-1}^0}{h_2^2} - T_0 (\beta_{11} \frac{u_{i+1,j}^1 - u_{i-1,j}^1 - u_{i+1,j}^{-1} + u_{i-1,j}^{-1}}{4h_1\tau} + \beta_{22} \frac{v_{i,j+1}^1 - v_{i,j-1}^1 - v_{i,j+1}^{-1} + v_{i,j-1}^{-1}}{4h_2\tau})) + T_{i,j}^0$$
(14)

Equation (6) can be written as:

$$a_i u_{i+1,j}^{n+1} + b_i u_{i,j}^{n+1} + c_i u_{i-1,j}^{n+1} = f_i \tag{15}$$

here $a_i = \frac{C_{1111}}{h_1^2}$, $b_i = -2(\frac{C_{1111}}{h_1^2} + \frac{\rho}{\tau^2})$, $c_i = \frac{C_{1111}}{h_1^2}$ and

$$f_i = \rho \frac{-2u_{i,j}^n + u_{i,j}^{n-1}}{\tau^2} - (C_{1122} + C_{1212}) \frac{v_{i+1,j+1}^n - v_{i-1,j+1}^n - v_{i+1,j-1}^n + v_{i-1,j-1}^n}{4h_1 h_2} - C_{1212} \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{h_2^2} + \beta_{11} \frac{T_{i+1,j}^n - T_{i-1,j}^n}{2h_1}$$

equation (7) can be written as:

$$a_i v_{i+1,j}^{n+1} + b_i v_{i,j}^{n+1} + c_i v_{i-1,j}^{n+1} = f_i \tag{16}$$

here $a_i = \frac{C_{1111}}{h_1^2}$, $b_i = -2(\frac{C_{1111}}{h_1^2} + \frac{\rho}{\tau^2})$, $c_i = \frac{C_{1111}}{h_1^2}$ and

$$f_i = \rho \frac{2v_{i,j}^n + v_{i,j}^{n-1}}{\tau^2} - (C_{1122} + C_{1212}) \frac{u_{i+1,j+1}^n - u_{i-1,j+1}^n - u_{i+1,j-1}^n + u_{i-1,j-1}^n}{4h_1 h_2} + C_{1212} \frac{v_{i+1,j}^n - 2v_{i,j}^n + v_{i-1,j}^n}{h_2^2} + \beta_{22} \frac{T_{i,j-1}^n - T_{i,j+1}^n}{2h_1}$$

equation (8) can be written as follows:

$$a_i T_{i+1,j}^{n+1} + b_i T_{i,j}^{n+1} + c_i T_{i-1,j}^{n+1} = f_i \tag{17}$$

here $a_i = \frac{\lambda_0}{h_1^2}$, $b_i = -\frac{2\lambda_0}{h_1^2} - \frac{C_\varepsilon}{\tau}$, $c_i = \frac{\lambda_0}{h_1^2}$ and

$$f_i = \lambda_{22} \frac{T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n}{h_2^2} - \lambda_{11} \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{h_1^2} - T_0 \left(\beta_{11} \frac{u_{i+1,j}^{n+1} - u_{i-1,j}^{n+1} - u_{i+1,j}^{n-1} + u_{i-1,j}^{n-1}}{4h_1 \tau} + \beta_{22} \frac{v_{i,j+1}^{n+1} - v_{i,j-1}^{n+1} - v_{i,j+1}^{n-1} + v_{i,j-1}^{n-1}}{4h_2 \tau} \right) - C_\varepsilon \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\tau}$$

Equation (15) $u(x, y, t)|_{x=\ell_1} = u_0, u(x, y, t)|_{x=\ell_1} = \bar{u}_0$, with boundary conditions, equation (16) $v(x, y, t)|_{x=0} = v_0, v(x, y, t)|_{x=\ell_1} = \bar{v}_0$ with boundary conditions, equation (17) $T(x, y, t)|_{x=0} = T_1(t), T(x, y, t)|_{x=0} = T_2(t)$ with boundary conditions, is solved by the method of nets.

RESEARCH RESULTS

Test problem, input constants: Lyambda11, Lyambda22 - Heat well tensors; Beta11, Beta22 - Coefficients of volumetric thermal expansion in the first and second motion equations; C1111, C1122, C1212, C2222 -

parameters characterizing the body; R_o - body density; C_e - is the heat capacity at constant temperature; T_0 - body temperature; h_1 - is the height between node points on the X -axis. h_2 - is the height between the node points along the Y -axis; τ - the time interval of the pens; n - is the number of steps.

$\lambda_{11} - 0.5, \lambda_{22} - 0.3, \beta_{11} - 0.05, \beta_{22} - 0.09, C_{1111} - 0.75, C_{1122} - 0.91, C_{1212} - 0.9, C_{2222} - 0.89, R_o - 1.1, C_e - 3.4, T_0 - 5, h_1 - 0.1, h_2 - 0.1, \tau - 0.01, n - 10.$

The following results were obtained on the basis of constant values for the given equations.

A clear solution to the problem

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.102	0.186	0.252	0.294	0.306	0.289	0.243	0.174	0.088	0.000
0.000	0.186	0.348	0.475	0.556	0.583	0.552	0.468	0.337	0.174	0.000
0.000	0.252	0.475	0.652	0.764	0.802	0.761	0.646	0.468	0.243	0.000
0.000	0.294	0.556	0.764	0.898	0.943	0.896	0.761	0.552	0.289	0.000
0.000	0.306	0.583	0.802	0.943	0.992	0.943	0.802	0.583	0.306	0.000
0.000	0.289	0.552	0.761	0.896	0.943	0.898	0.764	0.556	0.294	0.000
0.000	0.243	0.468	0.646	0.761	0.802	0.764	0.652	0.475	0.252	0.000
0.000	0.174	0.337	0.468	0.552	0.583	0.556	0.475	0.348	0.186	0.000
0.000	0.088	0.174	0.244	0.289	0.307	0.294	0.253	0.186	0.102	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Approximate solution to the problem

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.106	0.189	0.253	0.293	0.304	0.285	0.238	0.168	0.081	0.000
0.000	0.189	0.349	0.475	0.554	0.579	0.547	0.462	0.331	0.168	0.000
0.000	0.254	0.475	0.650	0.760	0.797	0.755	0.640	0.462	0.238	0.000
0.000	0.293	0.554	0.760	0.892	0.937	0.890	0.755	0.547	0.285	0.000
0.000	0.304	0.579	0.797	0.937	0.985	0.937	0.797	0.579	0.304	0.000
0.000	0.285	0.547	0.755	0.890	0.937	0.892	0.760	0.554	0.293	0.000
0.000	0.238	0.462	0.640	0.755	0.797	0.760	0.650	0.475	0.254	0.000
0.000	0.168	0.331	0.462	0.547	0.579	0.554	0.475	0.350	0.190	0.000
0.000	0.082	0.169	0.239	0.286	0.305	0.294	0.255	0.190	0.107	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Based on these results, we can see the state of change of U, V, T in a two-dimensional square plate as follows.

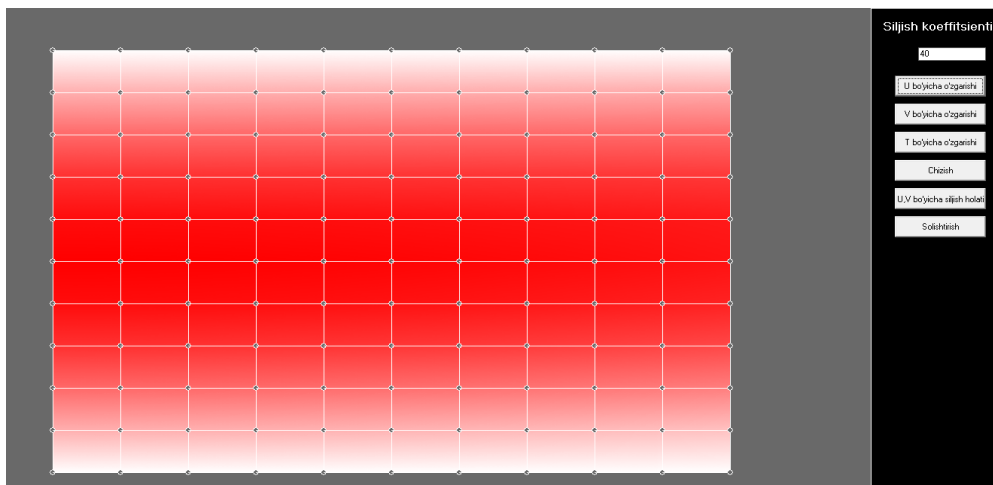


Figure 1. The state of change of U relative to the X axis

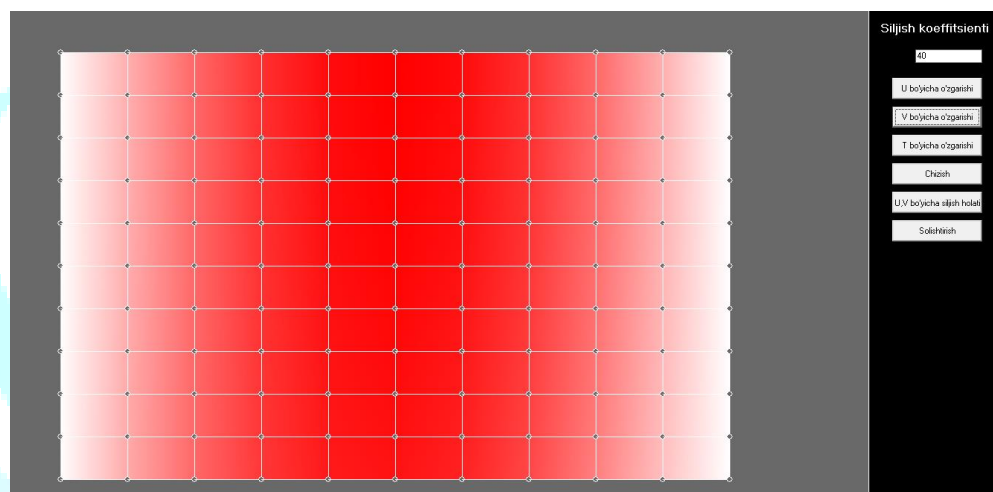


Figure 2. The state of change of V relative to the Y axis

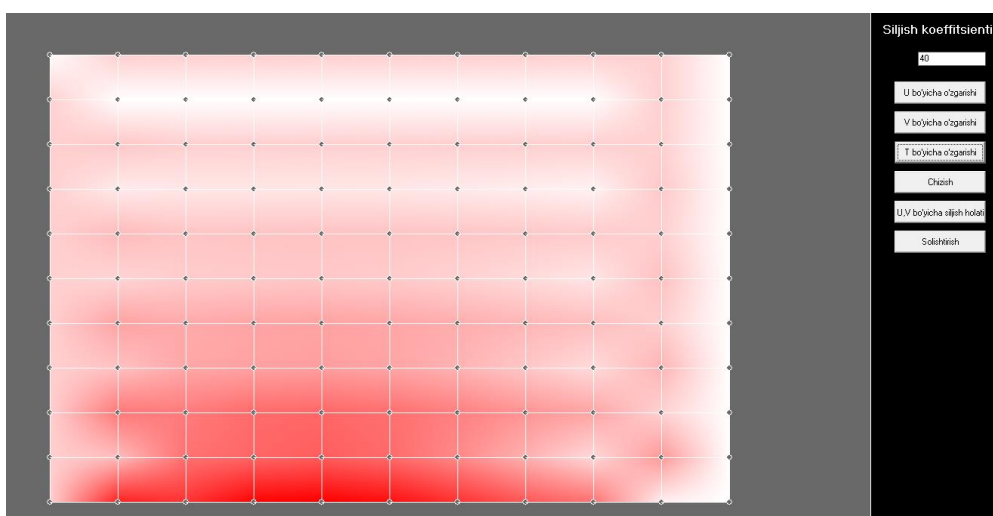


Figure 3. The state of impact of T on a square plate

CONCLUSION

In conclusion, mathematical models of many problems encountered in practice are brought to the study of thermoelastic or thermoplastic related and unrelated problems.

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