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Use of Mass Center to Prove Inequalities

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Annotation: This paper shows the most commonly used inequalities, in particular the ways to prove the Cauchy inequality using the center of mass. Emphasis is placed on the possibility of proving inequalities by geometric methods.

Keywords: Function, convex, concave, Jensen inequality, Cauchy inequality.

Depression and convexity of the function. Suppose that $f(x)$ the function is (a,b) given by $x_1, x_2 \in (a,b)$ and $x_1 < x_2$ for.

If we $f(x)$ say that a graph of a function is a straight $y = l(x)$ line passing through points, $(x_1, f(x_1))$ it is $(x_2, f(x_2))$ as follows

$$l(x) = \frac{x_2 - x}{x_2 - x_1} f(x_1) + \frac{x - x_1}{x_2 - x_1} f(x_2)$$

will be

Definition 1. For you to $(x_1, x_2) \subset (a,b)$ be located at $\forall x \in (x_1, x_2)$ any interval

$$f(x) \le l(x) \quad (f(x) < l(x))$$

If so, $f(x)$ the function is (a,b) called a fixed function.

Definition 2. For you to be $(x_1, x_2) \subset (a,b)$ located at any $\forall x \in (x_1, x_2)$ interval

$$f(x) \ge l(x) \quad (f(x) > l(x))$$

is called $f(x)$ a convex (a,b) (rigid convex) function.

Graphs of concave and convex functions are shown in Figure 1 (2).

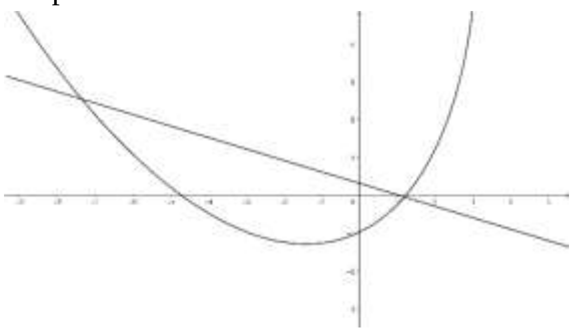

Figure 1

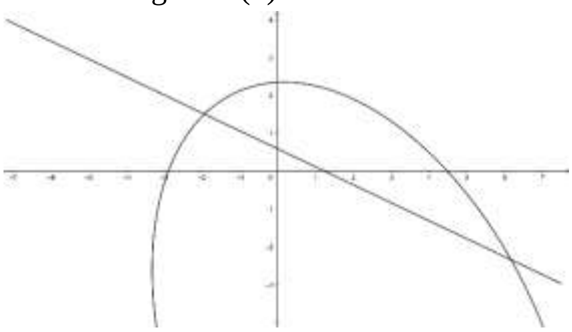

Figure 2

1- Theorem. Suppose that is given $f(x)$ by a function (a,b) that has a second-order product. In $f(x)$ order for a function (a,b) to be concave, it must be appropriate $f''(x) \ge 0$, $(f''(x) > 0)$ and sufficient.

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2- Theorem. Suppose that is $f(x)$ given by (a,b) a function that has a second-order product. In order $f(x)$ for a function to be convex (a,b) (rigidly convex), it must be $f''(x)\le 0$, $(f''(x)<0)$ appropriate and sufficient.

The center of mass of an object is a point defined relative to an object or system of bodies. The center of mass is an imaginary point at which the masses of all parts of a system of bodies appear to be concentrated.

The center of mass of simple homogeneous (consisting of only one substance) geometric shapes coincides with their geometric center. For example, the center of mass of a homogeneous disk is in the center of the circle that forms it. Sometimes the center of mass of an object does not belong to it. For example, the center of mass of a ring is in the middle of it, and there is no material at that point.

The coordinates of the center of mass of the bodies are found from the following formula.

$$(x,y)=\left(\frac{m_1x_1+m_2x_2+...+m_nx_n}{m_1+m_2+...+m_n};\frac{m_1y_1+m_2y_2+...+m_ny_n}{m_1+m_2+...+m_n}\right)$$

Let us now consider the following inequality.

Yensen inequality. Let be $f:(a,b)\to R$ a convex function. In that case $x_j\in(a,b)$ $(j=1,...,n)$ all s and

$$\lambda_1+\lambda_2+...+\lambda_n=1$$

for arbitrary numbers satisfying the equation

$$f(\lambda_1x_1+\lambda_2x_2+...+\lambda_nx_n)\ge\lambda_1f(x_1)+\lambda_2f(x_2)+...+\lambda_nf(x_n) \tag{1}$$

$$(f(\lambda_1x_1+\lambda_2x_2+...+\lambda_nx_n)\le\lambda_1f(x_1)+\lambda_2f(x_2)+...+\lambda_nf(x_n)) \tag{2}$$

inequality is appropriate.

We prove this inequality using the center of mass.

Proof. Let's look at $y=f(x)$ the points with abscissa of $x_1,x_2,...,x_n$ the May function. We place $A_1,A_2,...,A_n$ mass loads at these points. The center of mass $m_1,m_2,...,m_n$ of these loads is at the following point.

$$\left(\frac{m_1x_1+m_2x_2+...+m_nx_n}{m_1+m_2+...+m_n};\frac{m_1f(x_1)+m_2f(x_2)+...+m_nf(x_n)}{m_1+m_2+...+m_n}\right)$$

$A_1,A_2,...,A_n$ since the points lie on the graph of the convex function, the center of their masses also lies on the graph. This means that the ordinate of the center of mass is not smaller than the ordinate of the point with this abscissa.

$$f\left(\frac{m_1x_1+m_2x_2+...+m_nx_n}{m_1+m_2+...+m_n}\right)\le\frac{m_1f(x_1)+m_2f(x_2)+...+m_nf(x_n)}{m_1+m_2+...+m_n} \tag{3}$$

If we $\lambda_k=\dfrac{m_k}{m_1+m_2+...+m_n}$ $k=1,2,...,n$ assume that (1) inequality is as follows.

$$f\left(\lambda_1x_1+\lambda_2x_2+...+\lambda_nx_n\right)\le\lambda_1f(x_1)+\lambda_2f(x_2)+...+\lambda_nf(x_n)$$

here $\lambda_1+\lambda_2+...+\lambda_n=1$

Let us now consider some examples that can be proved using the Jensen inequality.

For example. prove the $e^{\frac{x+y}{2}} \leq \frac{e^x + e^y}{2}$ inequality.

We prove $y = e^x$ this inequality using Jensen's inequality. Since $y'' = e^x > 0$ this function is a subscript function, then inequality (2) holds.

$$f\left(\frac{1}{2}x + \frac{1}{2}y\right) \leq \frac{1}{2}f(x) + \frac{1}{2}f(y)$$

$$e^{\frac{x+y}{2}} \leq \frac{1}{2}e^x + \frac{1}{2}e^y$$

will be

Cauchy inequality about averages.

For arbitrary $a_1, a_2, ..., a_n$ positive numbers

$$\sqrt[n]{a_1 a_2 ... a_n} \leq \frac{a_1 + a_2 + ... + a_n}{n}$$

The inequality is reasonable, ie the geometric mean is not greater than the arithmetic mean.

Proof: This, let's $f(x) = \ln x$, $x > 0$ look at the function,

$$f''(x) < -\frac{1}{x^2}$$

The function $f(x)$ is convex. Therefore, according to formula (1), the following inequality holds.

$$\ln\left(\sum_{i=1}^{n} \frac{1}{n} a_i\right) \geq \sum_{i=1}^{n} \frac{1}{n} \ln a_i \quad (4)$$

can be written in the form. Inequality has been proven.

$$\sqrt[n]{a_1 a_2 ... a_n} \leq \frac{a_1 + a_2 + ... + a_n}{n}$$

Example: If $a, b, c > 0$ and, $a + b + c = 1$ and,

$$\left(a + \frac{1}{a}\right)^{10} + \left(b + \frac{1}{b}\right)^{10} + \left(c + \frac{1}{c}\right)^{10}$$

Find the smallest value of the expression.

Let's first look at the following function in the interval.

$$f(x) = \left(x + \frac{1}{x}\right)^{10}$$

is a second-order product of this function

$$f'(x) = 10\left(1 - \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)^9$$

$$f''(x) = 10\left(\frac{2}{x^3}\right)\left(x + \frac{1}{x}\right)^9 + 90\left(1 - \frac{1}{x^2}\right)^2\left(x + \frac{1}{x}\right)^8$$

it is clear that the function is sunk in the interval.

According to (2)

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$$f\left(\frac{a+b+c}{3}\right) \le \frac{1}{3}f(a)+\frac{1}{3}f(b)+\frac{1}{3}f(c)$$

appropriate, that is

$$\frac{1}{3}\left(\left(a+\frac{1}{a}\right)^{10}+\left(b+\frac{1}{b}\right)^{10}+\left(c+\frac{1}{c}\right)^{10}\right) \ge \left(\frac{a+b+c}{3}+\frac{3}{a+b+c}\right)^{10}=\left(\frac{10}{3}\right)^{10}$$

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$$\left(a+\frac{1}{a}\right)^{10}+\left(b+\frac{1}{b}\right)^{10}+\left(c+\frac{1}{c}\right)^{10} \ge \frac{10^{10}}{3^9}$$

As long as the smallest value $\frac{10^{10}}{3^9}$ of the expression is equal to.

In short, if we look at the functions geometrically, we get some important results in mind.

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