RESEARCH PARKS
INTERNATIONAL JOURNAL ON ORANGE TECHNOLOGY
https://journals.researchparks.org/index.php/IJOT e-ISSN: 2615-8140 | p-ISSN: 2615-7071
Volume: 5 Issue: 5 | May 2023

Solving the System of Nonlinear Equations of a Thin-Walled Cylindrical Shell

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Keywords: cylindrical shell, a physical nonlinear equation, thin-walled shell, stressed-strained state, circular elastic shell.

Introduction. We write the system of nonlinear equations of a thin-walled cylindrical shell in the following form:

$$\left[1+\frac{2}{3}\frac{\xi^2}{r_2^2}\gamma_2\left(\frac{\partial U_{\theta.1}}{\partial z}\right)^2\right]\left(\lambda-4\frac{r_1^2+r_2^2}{r_1^2 r_2^2}\right)U_{\theta.1}=-\frac{2r_1^2}{\xi\mu\left(r_2^2-r_1^2\right)}f_{r\theta}^{(2)}$$

$$\left[1+\frac{2}{3}\frac{\xi^2}{r_2^2}\gamma_2\left(\frac{\partial U_{\theta.1}}{\partial z}+\frac{r_2^2}{\xi}\frac{\partial U_{\theta.0}}{\partial z}\right)^2\right]\left(\lambda U_{\theta.0}+\frac{8\xi}{r_1^2 r_2^2}U_{\theta.1}\right)=\frac{4}{\mu\left(r_2^2-r_1^2\right)}f_{r\theta}^{(2)}(z,t) \tag{1.1}$$

We formulate practical problems for the generated system of equations. In order to form the boundary and initial conditions, in the second chapter, we use the formulas that define the stressed-deformed state in the cross-sections of the stern, so

$$U_\theta(r,z,t)=\frac{\xi}{r}U_{\theta.1}+rU_{\theta.0} \tag{1.2}$$

$$\sigma_{r\theta}(r,z,t)=\mu\left\{1+\frac{2}{3}\gamma_2\left[\left(-\frac{2\xi}{r^2}U_{\theta.1}+\frac{\xi}{2}\lambda U_{\theta.1}\right)^2+\left(\frac{\xi}{r}\frac{\partial U_{\theta.1}}{\partial z}+r\frac{\partial U_{\theta.0}}{\partial z}\right)^2\right]\right\}\times$$
$$\times\left(-\frac{2\xi}{r^2}U_{\theta.1}+\frac{\xi}{2}\lambda U_{\theta.1}\right),\qquad r_1\le r\le r_2. \tag{1.3}$$

$$\sigma_{z\theta}(r,z,t)=\mu\left\{1+\frac{2}{3}\gamma_2\left[\left(\frac{\xi}{r}\frac{\partial U_{\theta.1}}{\partial z}+r\frac{\partial U_{\theta.0}}{\partial z}\right)^2+\left(\frac{\xi}{2}\lambda U_{\theta.1}-\frac{2\xi}{r^2}U_{\theta.1}\right)^2\right]\right\}\times$$
$$\times\left(\frac{\xi}{r}\frac{\partial U_{\theta.1}}{\partial z}+r\frac{\partial U_{\theta.0}}{\partial z}\right),\qquad r_1\le r\le r_2. \tag{1.4}$$

Let us assume that a time-varying kinematic torsion shock is applied to the free end of a circular elastic shell, one end of which is free, the other end is clamped, and a constant stress of $f_{r\theta}^{(2)}(z,t)=const$ applied to its outer

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surface. At the beginning, the shell is at rest and its initial torsional velocity is also zero. Determine the stressed-deformed state of the shell sections, taking into account the physical nonlinearity in the links between stress and deformation. Based on the obtained results, the effect of physical nonlinearity on the torsional vibrations of the circular elastic shell should be studied and analyzed.

According to the condition of the problem, its initial and boundary conditions have the following form:

Prerequisites conditions: $U_\theta\big|_{t=0}=0$, $\left.\dfrac{\partial U_\theta}{\partial t}\right|_{t=0}=0$ (1.5)

Boundary conditions: $U_\theta\big|_{z=0}=g(t)$, $U_\theta\big|_{z=l}=0$ (1.6)

Here U_θ - torsional displacement of the shell point; l- shell length; $g(t)$- is a given continuous and integrable function of time t. If expression (1.2) is taken into account, then for functions $U_{\theta.0}$ and $U_{\theta.1}$ expressing conditions (1.5) and (1.6) we can write as follows:

1) For the required $U_{\theta.1}(z,t)$ functions:

$t=0$ at $U_{\theta.1}(z,t)=0$, $\dfrac{\partial U_{\theta.1}(z,t)}{\partial t}=0.$

$z=0$ at $U_{\theta.1}(z,t)=\dfrac{r}{2\xi}g(t)$, $z=1$ at $U_{\theta.1}(z,t)=0.$ (1.7)

2) For the required $U_{\theta.0}(z,t)$ functions:

$t=0$ at $U_{\theta.0}(z,t)=0$, $\dfrac{\partial U_{\theta.0}(z,t)}{\partial t}=0.$

$z=0$ at $U_{\theta.0}(z,t)=\dfrac{r}{2\xi}g(t)$, $z=1$ at $U_{\theta.0}(z,t)=0.$ (1.8)

The second of these initial and boundary conditions, that is, conditions (1.8) should be used to solve the first equation of the system (1.1), and conditions (1.7) should be used to solve the second equation of the system (1.1).

The method of solving the equation: In the system of equations (1.1) above, we introduce dimensionless coordinates as follows

$$t=t^*\frac{l}{b};\ z=z^*l;\ U_{\theta.0}=U^*;\ U_{\theta.1}=\xi V^*.$$

After a number of calculations, we create the following system:

$$\left[1+A_{11}\left(\frac{\partial V}{\partial z}\right)^2\right]\left(\frac{\partial^2 V}{\partial t^2}-\frac{\partial^2 V}{\partial z^2}-A_{12}V\right)=A_{13};\qquad (1.9)$$

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$$\left[1+A_{21}\left(\frac{\partial V}{\partial z}+A_{22}\frac{\partial U}{\partial z}\right)^2\right]\left(\frac{\partial^2 U}{\partial t^2}-\frac{\partial^2 U}{\partial z^2}-A_{23}V\right)=A_{24}. \qquad (1.10)$$

here

$$A_{11}=\frac{2}{3}\frac{\xi^4\gamma_2}{r_2^2 l^2};\; A_{12}=\frac{4\left(r_1^2+r_2^2\right)l^2}{r_2^2 r_1^2};\; A_{13}=-\frac{2r_1^2\,l^2}{\xi^2\left(r_2^2-r_1^2\right)}\frac{f_{r\theta}^{(2)}}{G};$$

$$A_{21}=\frac{2}{3}\frac{\xi^4\gamma_2}{r_2^4};\; A_{22}=\frac{r_2^2}{\xi^2};\; A_{23}=\frac{8\xi^2 l^2}{r_1^2 r_2^2};\; A_{24}=\frac{4\,l^2}{\left(r_2^2-r_1^2\right)}\frac{f_{r\theta}^{(2)}}{G}.$$

By making the following substitutions in the equations (1.9) and (1.10) above

$$\frac{\partial^2 U}{\partial t^2}\approx\frac{U_i^{j+1}-2U_i^j+U_i^{j-1}}{\tau^2};\quad \frac{\partial U}{\partial z}\approx\frac{U_{i+1}^j-U_{i-1}^j}{2h};\quad \frac{\partial^2 U}{\partial z^2}\approx\frac{U_{i-1}^j-2U_i^j+U_{i+1}^j}{h^2},$$

$$\frac{\partial^2 V}{\partial t^2}\approx\frac{V_i^{j+1}-2V_i^j+V_i^{j-1}}{\tau^2};\quad \frac{\partial V}{\partial z}\approx\frac{V_{i+1}^j-V_{i-1}^j}{2h};\quad \frac{\partial^2 V}{\partial z^2}\approx\frac{V_{i-1}^j-2V_i^j+V_{i+1}^j}{h^2},$$

we construct the following system of algebraic equations:

a) Equation (1.9) and conditions (1.7):

$$V_i^{j+1}=2V_i^j-V_i^{j-1}+\left(\frac{\tau}{h}\right)^2\left(V_{i-1}^j-2V_i^j+V_{i+1}^j\right)+\tau^2 A_{12}V_i^j+\tau^2 A_{13}\Big/\left[1+A_{11}\left(\frac{V_{i+1}^j-V_{i-1}^j}{2h}\right)^2\right]; \qquad (1.11)$$

$$z=0 \text{ at } V_0^j=\frac{r_2}{2\xi}g(j\tau);\; z=1 \text{ at } V_{n+1}^j=0. \qquad (1.12)$$

$$t=0 \text{ at } V_i^0=0;\; \frac{V_i^1-V_i^0}{h}=0. \qquad (1.13)$$

b) in equation (1.10) and conditions (1.8):

$$U_i^{j+1}=2U_i^j-U_i^{j-1}+\left(\frac{\tau}{h}\right)^2\left(U_{i-1}^j-2U_i^j+U_{i+1}^j\right)+\tau^2 A_{23}V_i^j+$$

$$+\tau^2 A_{24}\Big/\left[1+A_{21}\left(\frac{V_{i+1}^j-V_{i-1}^j}{2h}+A_{22}\frac{U_{i+1}^j-U_{i-1}^j}{2h}\right)^2\right]; \qquad (1.14)$$

$$z=0 \text{ at } U_0^j=\frac{1}{2r_2}g(j\tau);\; z=1 \text{ at } U_{n+1}^j=0. \qquad (1.15)$$

$$t=0 \text{ at } U_i^0=0;\; \frac{U_i^1-U_i^0}{h}=0. \qquad (1.16)$$

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Here, choosing $g(t) = A\sin(\pi t)$ views, we solve the above algebraic equations together and determine results $U(z_i, t_j)$ and $V(z_i, t_j)$.

Calculation results: Calculations for the material of the circular cylindrical shell (Steel) – $G = 0.87 \cdot 10^5 MPa$; $\gamma_2 = -0.085 \cdot 10^6$; $\rho = 7850\ kg/m^3$ we get the values. We get the geometric dimensions as follows:

$\xi = 0.061\,m$; $r_1 = 0.975\xi$; $r_2 = 1.025\xi$; $l = 1\,m$; $A = 1 \cdot 10^{-3} m$; $f_{r\theta}^{(2)}/G = 2.93 \cdot 10^{-7}$

In the following pictures, torsion U_θ displacement and $\sigma_{z\theta}$, $\sigma_{r\theta}$ tress changes in the value of external force $f_{r\theta}^{(2)}/G = 2.93 \cdot 10^{-7}$ in different sections of the shell as a function of time, and stergen points in different values of time torsion U_θ displacement and $\sigma_{z\theta}$, $\sigma_{r\theta}$ stress change in the value of external force $f_{r\theta}^{(2)}/G = 2.93 \cdot 10^{-7}$ depending on the coordinate graphs of changes are presented.

From the graphs of displacement U_θ presented in Figure 1.1, it can be seen that at different values of time, displacement U_θ of torsion has a vanishing character in the coordinate, regardless of the value of the external force. Maximum values of displacement at different values

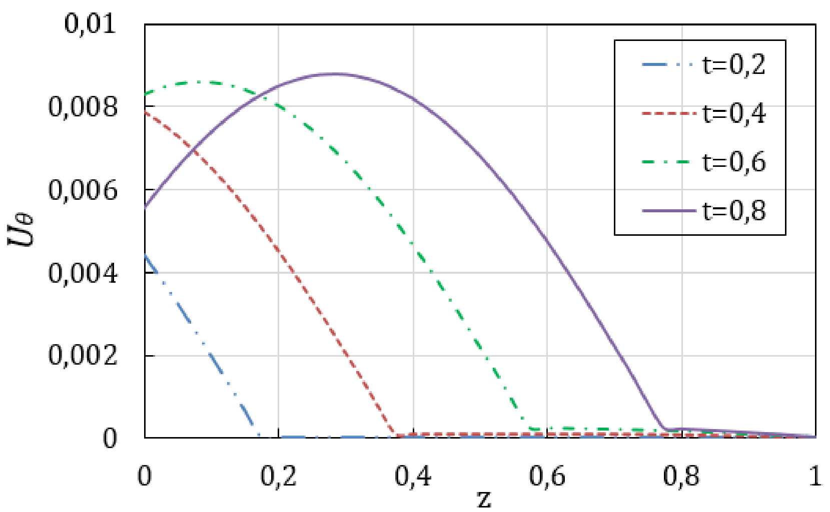

Figure 1.1. Coordinate-dependent variations of the torsional displacement of a thin-walled shell U_θ at $t = 0{,}2; 0{,}4; 0{,}6; 08$ points in time..

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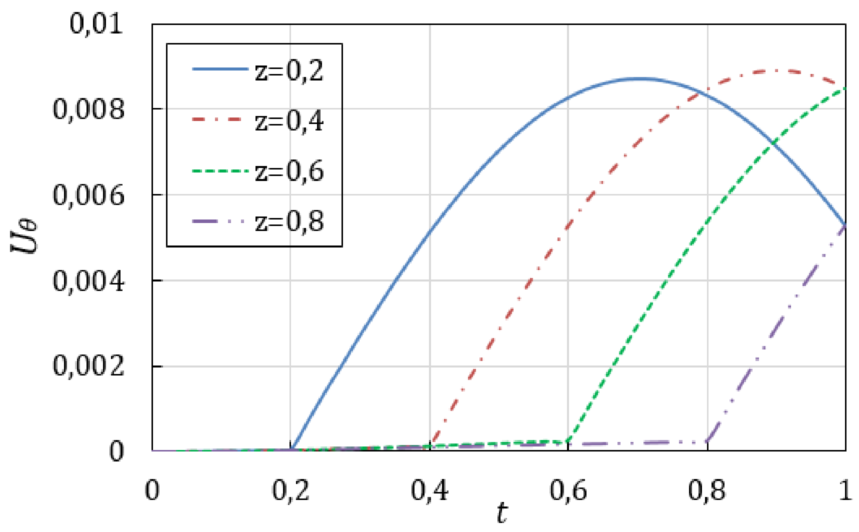
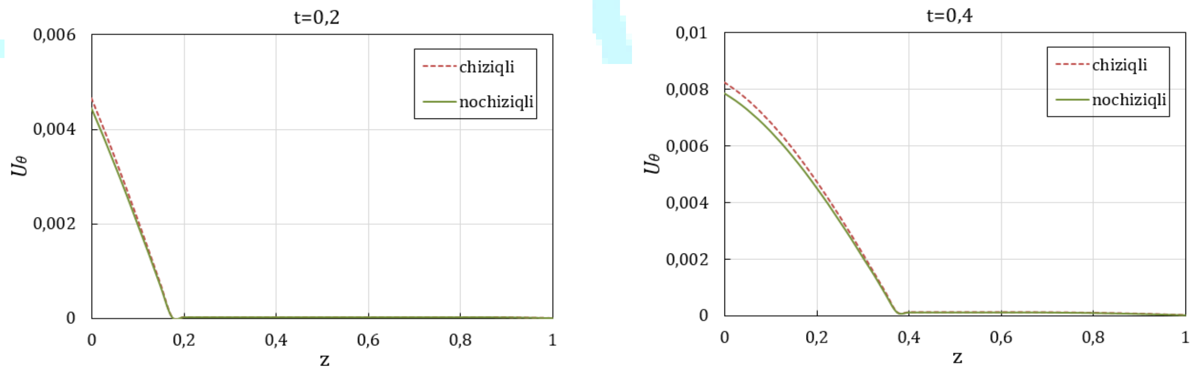


Figure 1.2. Thin-walled shell U_θ Time-dependent variations of torsional displacement in different sections of the shell.

the maximum values of the amplitude are the same. This effect is due to the elasticity of the material.

Graphs of time-dependent changes of displacement U_θ in different sections of the shell at the value of $f_{r\theta}^{(2)}/G = 2.93\cdot 10^{-7}$ external force (Fig. 1.2) are presented. It can be seen from the graphs that the excitation of the shell point changes with a sinusoidal regularity starting from the time when the excitation wave reaches the section. The graphs of torsion U_θ migration presented in Figures 1.3 and 1.4 also confirm the above conclusions.



a) b)

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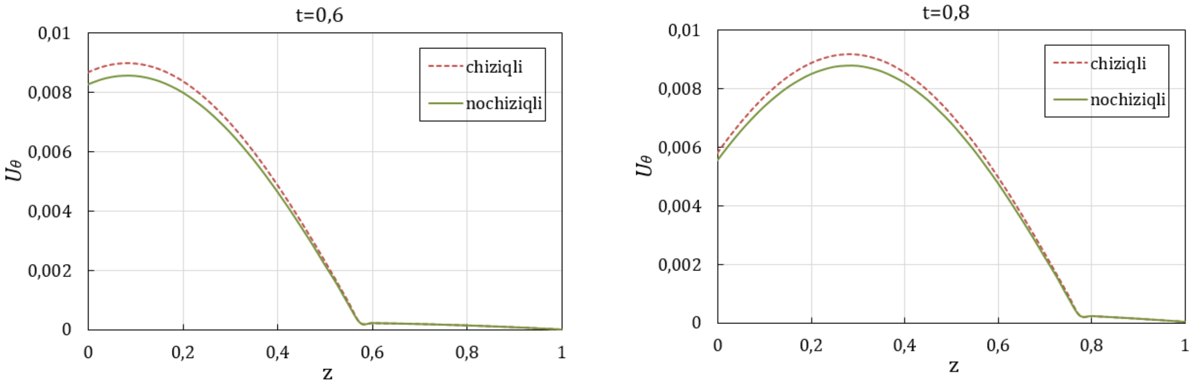
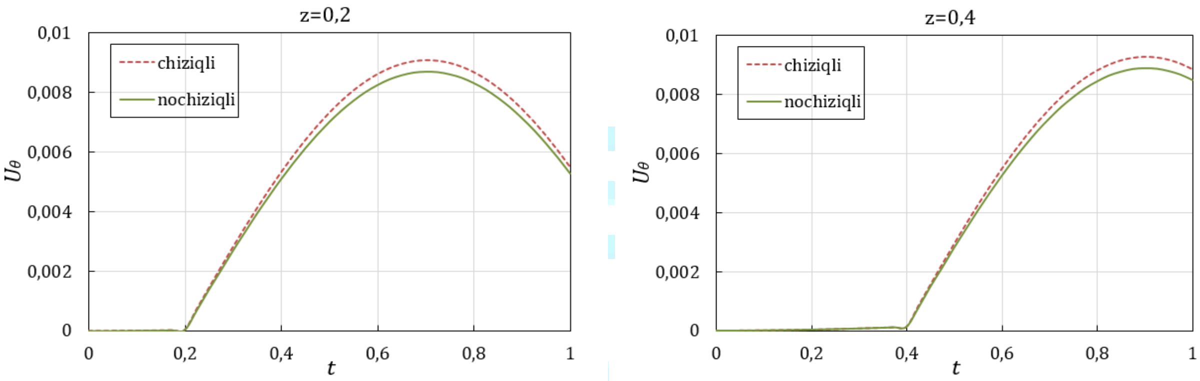


c) d)

Figure 1.3. Torsional displacement graphs for nonlinear and linear cases of thin-walled shell are compared.



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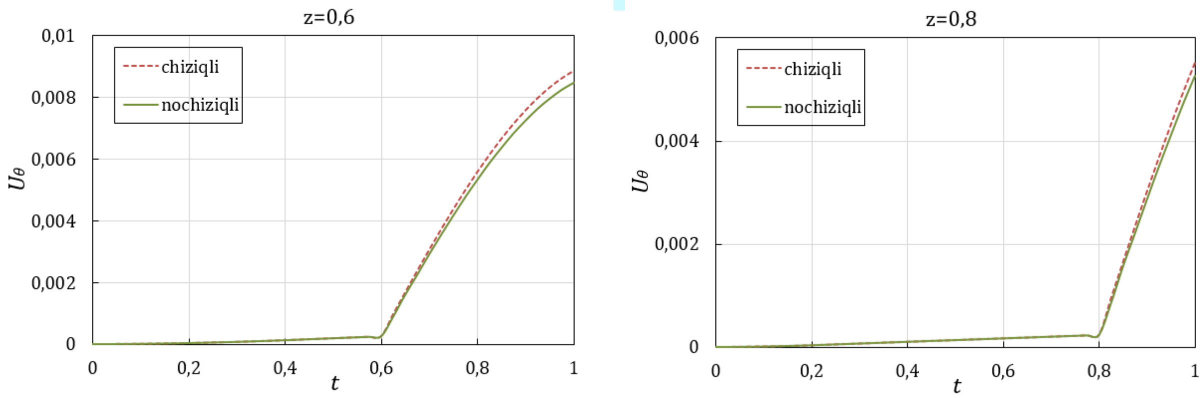

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Conculation. In conclusion, it can be said that the graphs of the voltages of test $\sigma_{z\theta}$ and $\sigma_{r\theta}$ show wave changes with $\sigma_{z\theta}$ relatively quiet, and $\sigma_{r\theta}$ with sharp jumps.

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