

# Unloaded plates free vibrations, supported by elastic thin-walled rods

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**Abstract-** The paper regarded free vibrations of a plate elastically clamped at  $y = \pm \frac{b}{2}$  edges with thin-walled open profile rods and pivotally supported on  $x = 0, a$  edges.

**Keyword:** Compressed, stretched, free vibrations, frequency, elastic pinching, hinged, torsional, bending, reinforced, non-reinforced, edges, stiffness.

## INTRODUCTION

**Problem statement and literature review.** It is known that the reinforced plates' free vibrations, taking into account the reinforcing ribs torsion, have been studied at present. In particular, this applies to plates reinforced with thin-walled open profile rods. Taking into account the constrained torsion specificity of thin-walled ribs (as will be shown below) significantly affects the "plate-rib" system stiffness parameters and, as a consequence, its natural vibrations frequencies.

The literature [1-3,6,7] describes the following drawing method up boundary conditions along the plate contact line with the reinforcing ribs. The loads transferred by the plate to the ribs (rods) are considered equal, but opposite in direction to the forces in the corresponding plate sections. Then, kinematic conditions for displacements equality at the contact points of the reinforcing rods with the plate are introduced into these force conditions.

In publications [4-5], a technique is proposed for drawing up refined boundary conditions on the plate conjugation line with the rod, which makes it possible to take into account the warping constraint the end sections of the ribs. In this case, the warping constraint degree is taken into account by some  $d_k$  parameter. This parameter is included in the expression

for the generalized coefficient of elastic pinching of  $t_k$  plate edges and depends not only on the geometric and mechanical characteristics of the bar and the plate, but also on the force transmitted to the edge and on the plate buckling form.

**Solution method.** As you know, a closed solution to the free vibrations problem of a rectangular plate can be obtained in single trigonometric series only when two parallel edges are hinged while the other two edges can be fixed in an arbitrary way which corresponds to M. Levy solution.

To derive the equation for plate free vibrations frequencies, resiliently clamped at  $y = \pm \frac{b}{2}$  edges and pivotally supported on the  $x = 0, a$  edges (Fig. 1).

Let us write down the well-known differential problem equation

$$D\nabla^2\nabla^2w + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \quad D = \frac{E\delta^3}{12(1-\mu^2)}, \quad (1)$$

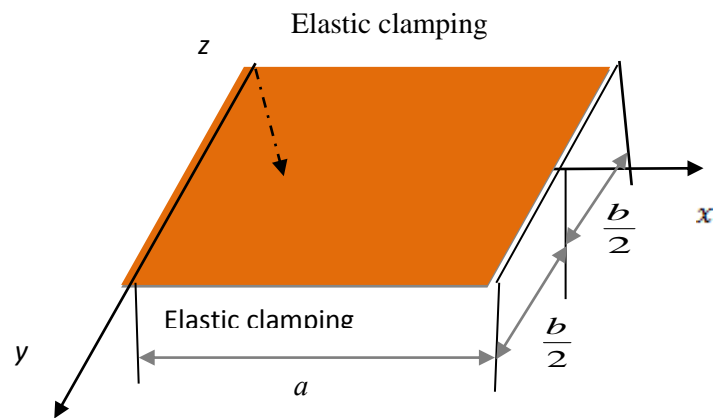


Fig.1 Free vibrations of a plate reinforced with thin-walled rods.

Considering harmonic vibrations, the deflection function can be represented in the form [1-3].

$$w(x, y, t) = w(x, y) \cos \omega t \quad (2)$$

where  $w(x, y) = \sum_{n=1,2..}^{\infty} f_n(y) \sin \lambda_n x$ ,  $\lambda_n = \frac{n\pi}{a}$ , (3)

Substituting (2) into (1), we obtain the equation for  $f_n(y)$  function

$$f_n^{IV} - 2\lambda_n^2 f_n'' + \left( \lambda_n^4 - \omega^2 \frac{\rho h}{D} \right) f_n = 0, \quad (4)$$

whose solution can be presented in the form

$$f_n(y) = C_1 ch \alpha_n y + C_2 sh \alpha_n y + C_3 \cos \beta_n y + C_4 \sin \beta_n y \quad (5)$$

where  $\alpha_n = \pm \sqrt{\omega_*^2 + \lambda_n^2}$ ,  $\beta_n = \pm \sqrt{\omega_*^2 - \lambda_n^2}$ ,

$$\omega_* = \omega \sqrt{\frac{\rho h}{D}} \quad (6)$$

When writing (6), it was accepted  $\omega_* > \lambda_n^2$ .

To determine the constants in (5), we use the boundary conditions [4-5].

$$f_n' = \frac{(\mu \lambda_n^2 f_n - f_n'')}{t_k \lambda_n^2 b}; \quad f_n(\pm b/2) = 0, \quad (7)$$

assuming first that the reinforced the plate edges are elastically restrained, but rigidly supported.

We will also assume that the reinforcing rods and their attachment to the plate are the same. This circumstance determines the same conditions for elastic edges clamping and allows using the plate symmetry curvature about the x axis.

Let us first consider vibration modes symmetric about this axis. Moreover, in (5) we can put  $C_2 = C_4 = 0$ .

Subordinating function (5) to boundary conditions (7), we arrive at the following system of two homogeneous equations for the constants  $C_1$  and  $C_3$ :

$$\begin{aligned} C_1 ch \xi + C_3 \cos \eta &= 0, \\ C_1 (2t_k \lambda_n^2 b^2 \xi sh \xi - \mu \lambda_n^2 b^2 ch \xi + 4\xi^2 ch \xi) - \\ - C_3 (2t_k \lambda_n^2 b^2 \eta \sin \eta + \mu \lambda_n^2 b^2 \cos \eta + 4\eta^2 \cos \eta) &= 0 \end{aligned} \quad (8)$$

In (8) denoted  $\xi = \frac{\alpha_n b}{2}$ ,  $\eta = \frac{\beta_n b}{2}$  (9)

Since the system trivial solution (8) is not in interest, we equate this system determinant to zero, expanding which we obtain:

$$(\eta \sin \eta + \xi th \xi \cos \eta) t_k \lambda_n^2 b^2 + 2(\xi^2 + \eta^2) \cos \eta = 0 \quad (10)$$

The relationship between  $\xi$  and  $\eta$ , as well as the formula for free vibrations frequency of the plate follow from (6):

$$\xi^2 - \eta^2 = \frac{1}{2} \lambda_n^2 b^2, \quad \omega_* b^2 = 2(\xi^2 + \eta^2) \quad (11)$$

The  $\xi$  and  $\eta$  values, satisfying equation (10), together with (11) determine the frequency symmetric vibration modes spectrum of the plate. The elastic pinching degree influence of the edges reinforced with ribs on the vibration frequencies and modes is determined by the dimensionless  $t_k$  coefficient,

$$t_k = \frac{k_1^2 C_k}{k^2 d_k} \left( \frac{\lambda_n^2}{k_1^2} - 1 \right), \quad C_k = \frac{GJ_k}{Db} \quad (12)$$

For skew-symmetric vibration modes in (5) should be taken  $C_1 = C_3 = 0$ . Submitting, as before, function (5) to boundary conditions (7) and repeating the previous reasoning, we arrive at a frequency form equation

$$(\xi \sin \eta - \eta th \xi \cos \eta) t_k \lambda_n^2 b^2 - 2(\xi^2 + \eta^2) h \xi \sin \eta = 0 \quad (13)$$

The notation included in (13) is given by expressions (9) and (12.)

Fig. 2 shows the dimensionless frequency parameter  $\omega_* b^2$  dependences on the  $C_k = GJ_k/Db$  coefficient constructed for various the flexural-torsional values characteristic to  $ka$  according to equations (10) and (13) for the lowest symmetric and skew-symmetric vibrations forms. In this case, the solid lines correspond to the free warping of the reinforcing ribs ends, and the dashed lines correspond to its complete constraint.

It follows from the graphs that a relatively elastic pinching degree small of the plate edges ( $C_k \approx 0,5 \div 1,5$ ) entails a significant increase in the frequency level in comparison with the hinged support case. The graphs also lead to the natural conclusion that different boundary conditions for the ends deplanation of the reinforcing rods have a rather significant effect on the elastic pinching degree of the plate edges. An increase in the frequency parameter (up to 10%) is explained by an increase in the "plate-rib" overall rigidity system.

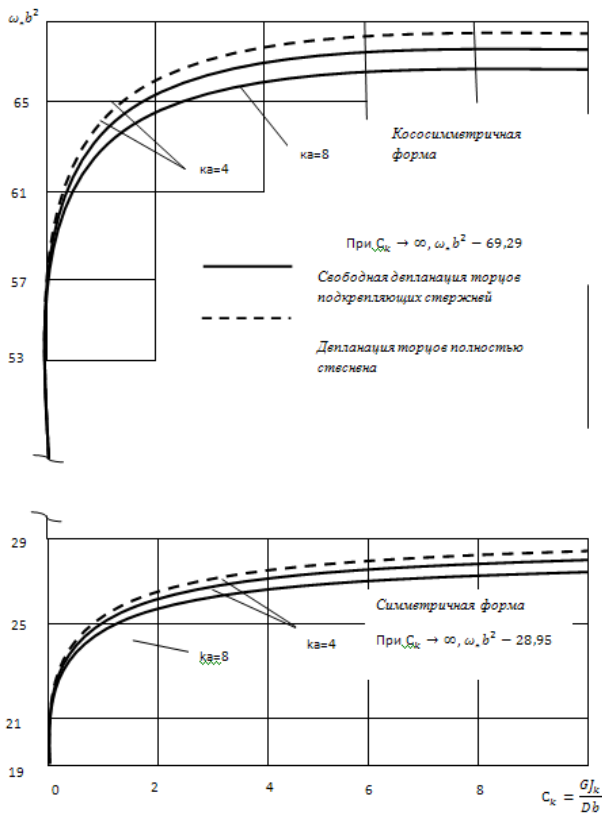


Fig. 2  $w b^2 \sqrt{\rho h / D}$  frequency parameter dependence on the degree elastic  $y = \pm b / 2$  edges pinching.

The graphs also lead to the natural conclusion that different boundary conditions for the reinforcing rods' ends deplanation have a rather significant effect on the elastic pinching degree of the plate edges. An increase in the frequency parameter (up to 10%) is explained by an increase in the "plate-rib" overall rigidity system.

Thus, taking into account the constrained torsion specificity in the reinforcing thin-walled ribs makes it possible to fully reveal the reserves for increasing the bending reinforced plate stiffness.

### Conclusion.

1. The generalized dimensionless elastic pinching coefficients and edges elastic support of the plate presented in the work take into account not only the mechanical and plate geometric parameters and thin-walled rods, but also different boundary conditions at the latter ends, a detailed attaching way the rods to the plate, as well as the longitudinal forces influence transmitted to the ribs. An essential feature

of these coefficients is their dependence on the half-wave' number of the plate curvature.

2. It is shown that taking into account the constrained torsion in the reinforcing ribs not only significantly increases the load critical parameters, but can also qualitatively change the curvature shape of the middle surface.

3. The warping restriction of the reinforcing ribs ends, as well as the elastic discrete warping presence bonds located along the rod axis, significantly increases the "plate - ribs" overall rigidity system and its natural vibrations frequency in comparison with free warping case of end sections and the warping connections absence.

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